Credit Shocks, Endogenous Lending Standards and Macroeconomic Dynamics

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Abstract

What are the effects of a credit shock in an environment featuring endogenously-persistent lending relationships between bank and firms and an endogenously-evolving lending standard? This paper answers this question in a DSGE model in which firm run by entrepreneurs borrow from bank who compete on both interest rates and collateral requirements. A credit shock in this model leads to an increase in replayment probability of loans at impact and a higher amplification of macroeconomic volatility compared to a model that does not feature lending relationships. Further, higher volatility and persistence of credit shock leads to greater macroeconomic volatility in presence of bank-firm lending relationships.

Keywords: Credit Shocks, Loan-to-Deposit ratio, Lending Standards, Macroeconomic Fluctuations

JEL Classification: E32, E44

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1 Introduction

What are the effects of a credit shock in a model in which borrowers form endogenously-persistent lending relationships with lenders and banks compete on both interest rates and collateral requirements? What implications does a credit shock have for lending standards and the resulting economic dynamics? This paper answers these questions in a simple model in which banks raise deposits from households and make loans to entrepreneurs who develop endogenously-persistent lending relationships with banks. A credit shock in this model, defined as a sudden increase in bank loans relative to their deposit, leads to an amplification of macroeconomic volatility. These effects are stronger in presence of lending relationships are increasing in volatility and persistence of credit shock.

The contribution of this paper is to show that a positive credit shock can lead to an increased amplification of macroeoconomic volatility when bank-firm lending relationships are taken into account. A corollary of this finding is that a model that assumes away credit relationships may underestimate the effects and implications of a credit shock.

A number of studies (Ongena and Smith, 2000; Kosekova, Maddaloni, Papoutsi, and Schivardi, 2023) have highlighted presence of lending relationships between borrowers and lenders. However, to the best of my knowledge, no work has examined the implications of a positive credit shock in a model that takes bank-firm credit relationships seriously. A recent work by Sharma (2023b) makes progress in this direction but it focuses only on bank competition on interest rates and abstracts from bank lending standards and how a positive credit shock might affect it and what implications it might have for the larger economy in such a model. This paper fills this gap. The banks in this model compete not only on interest rates but also collateral requirements. In the equilibrium, it gives rise to an endogenously-evolving lending standars. A positive credit shock in this model leads to an elevated oprobability of loan repayment before it falls briefly and returns to its equilibrium. All the macroecnomic variables in this model show more volatility than in a model which does not feature any lending standard. Another paper this work is connected to is Sharma (2023e) who examines changes in lending conditions by looking at state dependence in loan-to-value (LTV) shocks. That paper, however, does not consider bank-firm lending relationships or credit shocks.

This paper is related to the literature on lending relationships. Other notable contributions include, among others, Aliaga-Díaz and Olivero (2010), Ravn (2016), Airaudo and Olivero (2019) and Shapiro and Olivero (2020) and Sharma (2023c,a,d). None of these papers study the effects of a credit shock, modelled as a sudden rise in bank loans relative to deposits, in an environment of lending relationships. None of these papers, though, examine the effects of a positive credit shock on macroeocnomic activity. This paper also connects to another strand of literature that examines the effects of a credit shock on the macroeconomy. An example is Pesaran and Xu (2016). They examine of the effects of positive loan-to-demand shock in a model of firm default and abstract from presence of lending realtionships between borrowers and lenders. This paper, in the interest of simplicity, abstracts from firm default and focuses on implications of lending relationships for a positive credit shock.

2 Model

The paper features a Two Agent New Keynesian (TANK) model and bears resemblance to the setup in Iacoviello (2005), Liu, Wang, and Zha (2013) and Justiniano, Primiceri, and Tambalotti (2015). The departure from the models in these papers is inclusion of a formal financial sector and presence of lending relationships between firms and banks.

There are two types of agents. The first type of agents are (patient) households who consume, supply labor, make deposits with a bank and receive profits from the firms they own. The second type of agents are (impatient) entrepreneurs who consume non-durable consumption good and run firms in the economy. They are subject to a collateral constraint which limits their borrowing to a fraction of expected value of theit assets which include productive capital and (durable) land. The entrepreneurs borrow from banks and develop endogenously-persistent credit relationships with them. Lending relationships in this paper are modelled by using the deep habits framework developed first by Ravn, Schmitt-Grohé, and Uribe (2006) and used later in studying banking sector by Aliaga-Díaz and Olivero (2010), Airaudo and Olivero (2019) and Shapiro and Olivero (2020), among others. These banks raise deposits from households which is their only source of funding and lend them to entrepreneurs who combine them with productive capital to produce output. In what follows, I describe each agent's optimization problem.

2.1 Households

Households have the utility function of the following form:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^P\right)^t \left\{ \log\left(C_{i,t}^P - \gamma^P C_{i,t-1}^P\right) - \frac{N_{i,t}^{\nu}}{\nu} + \varsigma \log H_{i,t}^P \right\} \tag{1}$$

where $C_{i,t}^P$, $N_{i,t}^P$ and $H_{i,t}^P$ denote consumption, labor and housing respectively of the patient households, $\beta^P \in (0,1)$ is a discount factor and γ^P measures the degree of habit formation in consumption, ν is Frisch elasticity of labor supply and ς is a weight on housing. The superscript P denotes patient households. The household faces the following budget constraint

$$C_{i,t}^P + Q_t^H \left(H_{i,t}^P - H_{i,t-1}^P\right) + \int_0^1 D_{ik,t} \mathrm{d}k \leq W_t N_{i,t} + \int_0^1 \Pi_{ik,t} \mathrm{d}k + R_{t-1}^D \int_0^1 D_{ik,t-1} \mathrm{d}k \tag{2}$$

Here, Q_t^H is the price of one unit of housing in terms of consumption goods, W_t is the real wage and R_{t-1}^D is the gross risk-free interest rate on the stock of deposits $D_{ik,t-1}$ of household i in bank k at the end of period $t-1$. I assume housing does not depreciate. Profits obtained by household i from bank k are denoted by $\Pi_{ik,t}$. After imposing symetric equilibrium, FOCs of the households can be written as

$$\frac{1}{C_t^P - \gamma^P C_{t-1}^P} - \beta^P \mathbb{E}_t \frac{\gamma^P}{C_{t+1}^P - \gamma^P C_t^P} = \lambda_t^P \tag{3}$$

$$\beta^P \mathbb{E}_t \lambda_{t+1}^P = \frac{\lambda_t^P}{R_t^D} \tag{4}$$

$$\frac{\varsigma}{H_t^P} + \beta^P \mathbb{E}_t \left(\lambda_{t+1}^P Q_{t+1}^H\right) = \lambda_t^P Q_t^H \tag{5}$$

$$N_t^{\nu-1} = \lambda_t^P W_t^P \tag{6}$$

where λ_t^P is the Lagrange multiplier associated with household's budget constraint (2). One can combine household's first-order conditions with respect to consumption (3) and bank deposits (4) to obtain their Euler equation. Equation (5) describes household's Euler equation for housing and links today's housing price to the utility it provides plus the expected capital gain. Equation (6) describes household's consumption-lesiure tradeoff. First order conditions of the problem are derived in the Appendix A.

2.2 Entrepreneurs

Entrepreneur j maximizes the utility obtained from consuming the non-durable consumption goods

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^E\right)^t \log\left(C_{j,t}^E - \gamma^E C_{j,t-1}^E\right) \tag{7}$$

where β^E and γ^E are as defined above. I assume that entrepreneurs face a collateral constraint that limits the borrowing of each entrepreneur from each bank to a fraction of his assets

$$l_{jk,t}^E \leq \frac{1}{R_{k,t}^L} \theta_{k,t} a_{j,t}^E \tag{8}$$

Here, $l_{jk,t}^E$ denotes entrepreneur j's loan from bank k, expected value of entrepreneur's assets is $a_{j,t}^E$ and $R_{k,t}^L$ is the bank-specific lending rate. All entrepreneurs borrowing from bank k are subject to a loan-to-value (LTV) requirement $\theta_{k,t}$. In turn, $a_{j,t}^E$ is given by

$$a_{j,t}^E = \mathbb{E}_t \left(Q_{t+1}^H H_{j,t}^E + Q_{t+1}^K K_{j,t}\right) \tag{9}$$

In the above equation, Q_t^K denotes the value of installed capital in units of consumption goods, $K_{j,t}$ stock of capital and $H_{j,t}^E$ stock of housing.

Entrepreneurs have deep habits in banking relationships and and I let $x_{j,t}^E$ denote entrepreneur j's effective or habit-adjusted borrowing. Given the continuum of banks in the economy who compete under monopolistic competition, this can be written as

$$x_{j,t}^E = \left[\int_0^1 \left(l_{jk,t}^E - \gamma^L s_{k,t-1}^E\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k\right]^{\frac{\xi}{\xi-1}} \tag{10}$$

where stock of habits $s_{k,t-1}$ evolves according to

$$s_{k,t-1}^E = \rho_s s_{k,t-2}^E + (1-\rho_s)\, l_{k,t-1}^E \tag{11}$$

Here, $\gamma^L \in (0,1)$ denotes the degree of habit formation in demand for loans and $\rho_s \in (0,1)$ measures the persistence of this habits. The parameter ξ denotes of the elasticity of substitution between loans from different banks and is thus a measure of the market power of each individual bank.

Given his total need for financing $x_{j,t}^E$, each entrepreneur chooses $l_{jk,t}^E$ to solve the following problem

$$\min_{l^E_{jk,t}} \int_0^1 \Upsilon_{k,t} l^E_{jk,t} \mathrm{d}k \tag{12}$$

subject to collateral constraint (8) and his effective borrowing (10). Here, $\Upsilon_{k,t} \equiv R^L_{k,t} + \frac{\eta}{\theta_{k,t}}$ where the first term denotes the interest expenditure and the second term refers to value of pledged collateral. Entrepreneur j's optimal demand for loans from bank k is

$$l^E_{jk,t} = \left(\frac{\Upsilon_{k,t}}{\Upsilon_t}\right)^{-\xi} x^E_t + \gamma^L s^E_{k,t-1} \tag{13}$$

where $\Upsilon \equiv R^L_t + \eta \frac{1}{\theta_t}$ with $\theta_t = \left(\int_0^1 \theta^{1-\xi}_{k,t} \mathrm{d}k\right)^{\frac{1}{1-\xi}}$ representing the aggregate LTV ratio in the economy and $R^L_t \equiv \left[\int_0^1 (R^L_{k,t})^{1-\xi} \mathrm{d}k\right]^{\frac{1}{1-\xi}}$ the aggregate lending rate.

Production function of each entrepreneur is

$$Y_{j,t} = A_t \left(N_{j,t}\right)^{1-\alpha} \left\{\left(H^E_{j,t-1}\right)^{\phi} \left(K_{j,t-1}\right)^{1-\phi}\right\}^{\alpha} \tag{14}$$

where $Y_{j,t}$ is output, $N_{i,t}$ is labor input and $\alpha, \phi \in (0,1)$ are factor shares. TFP A_t follows the process

$$\log A_t = (1-\rho_A) \log A + \rho_A \log A_{t-1} + \sigma_A \epsilon_{A,t} \tag{15}$$

with iid innovation $\epsilon_{A,t}$ following a normal process with standard deviation σ_A where $A > 0$ and $\rho_A \in (0,1)$. The evolution of capital obeys the following law of motion

$$K_{j,t} = (1-\delta) K_{j,t-1} + \left[1 - \frac{\Omega}{2}\left(\frac{I_{j,t}}{I_{j,t-1}} - 1\right)^2\right] I_{j,t} \tag{16}$$

where $I_{j,t}$ is firm j's investment level, $\delta \in (0,1)$ the rate of depreciation of capital stock and $\Omega > 0$ is a cost adjustment parameter. The entrepreneur faces the following budget constraint

$$C^E_{j,t} + \int_0^1 R^L_{k,t-1} l^E_{jk,t-1} \mathrm{d}k \leq Y_{j,t} - W_t N_{j,t} - I_{j,t} - Q^H_t \left(H^E_{j,t} - H^E_{j,t-1}\right) + x^E_{j,t} + \Phi^E_t + \Psi^E_t \tag{17}$$

After imposing symmetric equilibrium, the FOCs of the entrepreneurs are

$$\lambda_t^E = \frac{1}{C_t^E - \gamma^E C_{t-1}^E} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C_{t+1}^E - \gamma^E C_t^E} \tag{18}$$

$$\lambda_t^E = \beta^E \mathbb{E}_t \lambda_{t+1}^E R_t^L + \mu_t^E R_t^L \tag{19}$$

$$W_t = (1-\alpha)\frac{Y_t}{N_t^P} \tag{20}$$

$$\lambda_t^E Q_t^H = \beta^E \mathbb{E}_t \left[\lambda_{t+1}^E \left(Q_{t+1}^H + \alpha\phi \frac{Y_{t+1}}{H_t^E}\right)\right] + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^H \tag{21}$$

$$\kappa_t^E = \alpha(1-\phi)\beta^E \mathbb{E}_t \left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t}\right) + \beta^E (1-\delta)\mathbb{E}_t \kappa_{t+1}^E + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^K \tag{22}$$

$$\lambda_t^E = \kappa_t^E \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \Omega \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + \beta^E \Omega \mathbb{E}_t \left[\kappa_{t+1}^E \left(\frac{I_{t+1}}{I_t}\right)^2 \left(\frac{I_{t+1}}{I_t} - 1\right)\right] \tag{23}$$

where μ_t^E, κ_t^E and λ_t^E are Lagrange multipliers associated with entrepreneur's collateral constraint (8), law of motion of capital (16) and entrepreneur's budget constraint (17). Entrepreneur's first order conditions with respect to consumption (18) and loans (19) may be combined to derive Euler equation for consumption for a collateral-constrained agent. Equation (20) describes entrepreneur's optimal demand for labor. Entrepreneur's Euler equation for land is described by (21) which relates its price today to its expected resale value tomorrow plus the payoff obatained by holding it for a period as given by its marginal producivity and its ability to serve as a collateral. Likewise, (22) is entrepreneur's Euler equation for capital and it links price of capital today to its price tomorrow and the expected payoff from keeping it for a period as given by its marginal productivity and its ability to serve as a collateral. Finally, entrepreneur's Euler equation for the investment is given by (23). All derivations of first order conditions are contained in Appendix A.

2.3 Banking Sector

Banks in this model accept deposits from patient households and make loans to entrepreneurs. Banks take the interest rate on deposits R_t^D as given. Each individual bank k chooses its lending rate $R_{k,t}^L$, its LTV ratio $\theta_{k,t}$ and its total amount of lending $L_{k,t}$. The link between lower credit standards and higher credit risk is given as

$$p_{k,t} = \Xi + \varpi\left(\theta_{k,t} - \overline{\theta}\right) \tag{24}$$

$p_{k,t}$ is bank-specific probability that a given loan is repaid and $\omega < 0$ measures the elasticity of this probability with respect to deviations of the bank's LTV ratio from its steady state level $\overline{\theta}$ which is same for all banks. Steady state repayment probability is given by $\Xi > 0$.

Each bank faces a standard trade-off when choosing its lending rate $R^L_{k,t}$. Profits of the bank k can be written as

$$
\begin{aligned}
\Pi_{k,t} &= \left[\Xi + \varpi\left(\theta_{k,t-1} - \overline{\theta}\right)\right] R^L_{k,t-1} L^E_{k,t-1} + \left[1 - \Xi - \varpi\left(\theta_{k,t-1} - \overline{\theta}\right)\right] \frac{L^E_{k,t-1}}{\int_0^1 L^E_{k,t-1} \mathrm{d}k} \tau \theta_{t-1} a^E_{t-1} \\
&\quad + \int_0^1 D_{ik,t} \mathrm{d}i - L^E_{k,t} - R^D_{t-1} \int_0^1 D_{ik,t-1} \mathrm{d}i \\
&= p_t R^L_{k,t-1} L^E_{k,t-1} + (1 - p_t) \frac{L^E_{k,t-1}}{\int_0^1 L^E_{k,t-1} \mathrm{d}k} \tau \theta_{t-1} a^E_{t-1} + \int_0^1 D_{ik,t} \mathrm{d}i - L^E_{k,t} - R^D_{t-1} \int_0^1 D_{ik,t-1} \mathrm{d}i
\end{aligned} \tag{25}
$$

With probability $p_{k,t-1}$, the bank receives its loan back with interest. With complementary probability $(1 - p_{k,t-1})$, the loan is not reapid in which case bank k receives a share of the liquidation value of the borrower's total collaterized assets with its share given by its total lending relative to total lending of all other firms. Their balance sheet follows the structure in Freixas and Rochet (2023). The balance sheet of bank k is

$$
L_{k,t} = \psi_t \int_0^1 D_{ik,t} \mathrm{d}i \tag{26}
$$

where $L_{k,t}$ denotes total loans made by bank k to all entrepreneurs, that is, $L_{k,t} \equiv \int_0^1 l^E_{jk,t} \mathrm{d}j$ and ψ_t is an exogenous shock which obeys the following law of motion

$$
\log \psi_t = (1 - \rho_\psi) \log \psi + \rho_\psi \log \psi_{t-1} + \sigma_\psi \epsilon_{\psi,t} \tag{27}
$$

. Each bank takes the demand for its loans as given

$$
L_{k,t} = \int_0^1 l^E_{jk,t} \mathrm{d}j = \int_0^1 \left[\left(\frac{\Upsilon_{k,t}}{\Upsilon_t}\right)^{-\xi} x^E_t + \gamma^L s^E_{k,t-1}\right] \mathrm{d}j \tag{28}
$$

Each bank chooses $L^E_{k,t}$, $\theta_{k,t}$ and $R^L_{k,t}$ to maximize its profits subject to (26) and (28). Considering a symmetric equilibrium in which all banks optimally choose the same LTV ratio and the same lending rate, the FOCs for banks' optimization problem are:

$$\varrho_t^E = \mathbb{E}_t q_{t,t+1}\left[p_{k,t}R_{k,t}^L + (1-p_{k,t})\frac{\tau\theta_t a_t^E}{\int_0^1 L_{k,t}^E \mathrm{d}k} - R_t^D + \gamma^L(1-\rho_s)\,\mathbb{E}_t\varrho_{t+1}^E\right] \tag{29}$$

$$\xi\varrho_t^E x_t^E \frac{\frac{\eta}{\theta_t}}{R_t^L\theta_t + \eta} = -\varpi\mathbb{E}_t q_{t,t+1}\left(R_t^L L_t^E - \tau\theta_t a_t^E\right) \tag{30}$$

$$\xi\varrho_t^E x_t^E \frac{\theta_t}{\theta_t R_t^L + \eta} = \mathbb{E}_t q_{t,t+1} p_{k,t} L_{k,t}^E \tag{31}$$

Derivation of all first order conditions have been relegated to Appendix A.

2.4 Aggregation and Market Clearing

Aggregate resource constraint of the economy is

$$C_t^P + C_t^E + I_t = Y_t \tag{32}$$

The clearing condition for the housing market is

$$H_t^P + H_t^E = H \tag{33}$$

where H is the total fixed supply of housing.

3 Equilibrium and Model Solution

The model is solved around its deterministic steady state using standard perturbation techniques (Adjemian, Bastani, Juillard, Karamé, Mihoubi, Mutschler, Pfeifer, Ratto, Rion, and Villemot, 2022). A period in the model is a quarter. The model is calibrated using parameter values standard in literature. The degree of habit formation is chosen to be 0.6 which is a common estimate in the literature (Smets and Wouters, 2007). The Frisch elasticity of labor supply ν is 1.01 while the weight on housing ς is set to 0.1 (Iacoviello, 2005).

The labor income share takes a standard value of 0.3 which yields a steady-state capital-output ratio of 1.15, consistent with US data (Liu, Wang, and Zha, 2013). The input share of land in production is close to the value estimated by Liu, Wang, and Zha (2013) and in line with the value used in Iacoviello (2005). The investment adjustment cost paramter is set to 1.85. The

Table 1: Parameter values

	Value	Description	Source/Target
β^P	0.995	Discount factor, patient households	Iacoviello (2005)
β^E	0.95	Discount factor, entrepreneurs	Iacoviello (2005)
γ^i, $i = \{P, E\}$	0.6	Habits in consumption	Ravn (2016)
ν	1.01	Frisch elasticity of labor	Iacoviello (2005)
ς	0.1	Weight on housing	Iacoviello (2005)
α	0.3	Non-labor share of production	Ravn (2016)
ϕ	0.1	Land share of non-labor input	Ravn (2016)
Ω	1.85	Investment adjustment cost parameter	Ravn (2016)
δ	0.0285	Capital depreciation rate	Ravn (2016)
τ	0.9432	Recovery rate of assets in liquidation	Ravn (2016)
Ξ	0.98	Steady state of repayment probability	Ravn (2016)
γ^L	0.72	Deep habit formation	Aliaga-Díaz and Olivero (2010)
ρ_s	0.93	Persistence of stock of deep habits	Ravn (2016)
ξ	230	Elasticity of substitution between banks	Ravn (2016)
ϖ	-1.5	Elasticity of credit risk	Ravn (2016)
η	0.05	Weight of collateral minimization desire	Ravn (2016)
ρ_A	0.95	Persistence of technology shock	Smets and Wouters (2007)
ρ_ψ	0.848	Persistence of credit shock	Pesaran and Xu (2016)
σ_A	0.0014	Standard deviation of technology shock	See Text
σ_ψ	0.011	Standard deviation of credit shock	Pesaran and Xu (2016)
ψ	1	Mean of loan-to-deposit ratio	See Text

available estimates range from close to 0 (Liu, Wang, and Zha, 2013) to above 26 (Christiano, Motto, and Rostagno, 2010). The rate of depreciation of capital is chosen to obtain a steady-state ratio of non-residential investment to output of slightly above 0.13 as consistent with US data (Beaudry and Lahiri, 2014). Following Liu, Wang, and Zha (2013), the recovery rate of assets in liquidation is calibrated to obtain an LTV ratio of 0.75 in steady state. The delinquency rate on commercial and industrial business loans in the US has fluctuated around an average close to 2% since mid 1990's. Using this, steady-state value of loan repayment probability Ξ is set to 0.98.

For parameters in the banking sector, I rely on Aliaga-Díaz and Olivero (2010). I set the deep habit parameter in lending γ^L to 0.72, only as the baseline and later vary it to capture in a transparent fashion how it affects credit shocks. Similarly, I set the autocorrelation parameter in stock of habits in lending ρ_s to 0.93 which is close to the value of 0.85 used by both Ravn, Schmitt-Grohé, and Uribe (2006) and Aliaga-Díaz and Olivero (2010). This gives a bank-firm relationship of 11 years (Petersen and Rajan, 1995). I take this as baseline and later vary it to study the effects of lending relationship persistence on transmission of credit shocks. Specifically, I consider the effects of credit shocks when stock of habits is such that after 10 years, stock of habits left is 0% and 10%. Setting $\gamma^L = 0$ shuts off deep habits in banking and setting $\rho = 0.86$ implies that after 44 quarters, the stock of habits is zero. I run simulations in which I gradually lower γ^L from its baseline value and examine impulse response functions. I also conduct experiments in which I consider two other autocorrelation parameters $\rho_s = 0.86$ and $\rho_s = 0.949$ denoting 0% and 10% stock of habit after 10 years, respectively. This allows me to investigate the impact of persistence of lending relationships on credit shocks. For elasticity of substitution between different loan varieties ξ, I pick the value as 230 which is close to the value of 190 used in Aliaga-Díaz and Olivero (2010) while Melina and Villa (2018) use a value of 427.

Elasticity of substitution between loans from different banks is calibrated so that interest rate spread between deposit and lending rates is 0.0168 in steady state (Aliaga-Díaz and Olivero, 2010). This implies an elasticity of substitution of 230 which is higher than elasticities of substituion usually employed in models of monopolistic competition in goods markets (Ravn, Schmitt-Grohé, and Uribe (2006) use a value of 5.3). Nevertheless, Aliaga-Díaz and Olivero (2010) argue that loans from different banks are likely to be much better substitutes than products of different firms in the goods markets, as also reflected in much smaller observed markups. This suggests that elasticity of substitution should indeed be much higher. In fact, Aliaga-Díaz and Olivero (2010) use an elasticity of substitution of 190 whereas Melina and Villa (2018) use a value of 427.

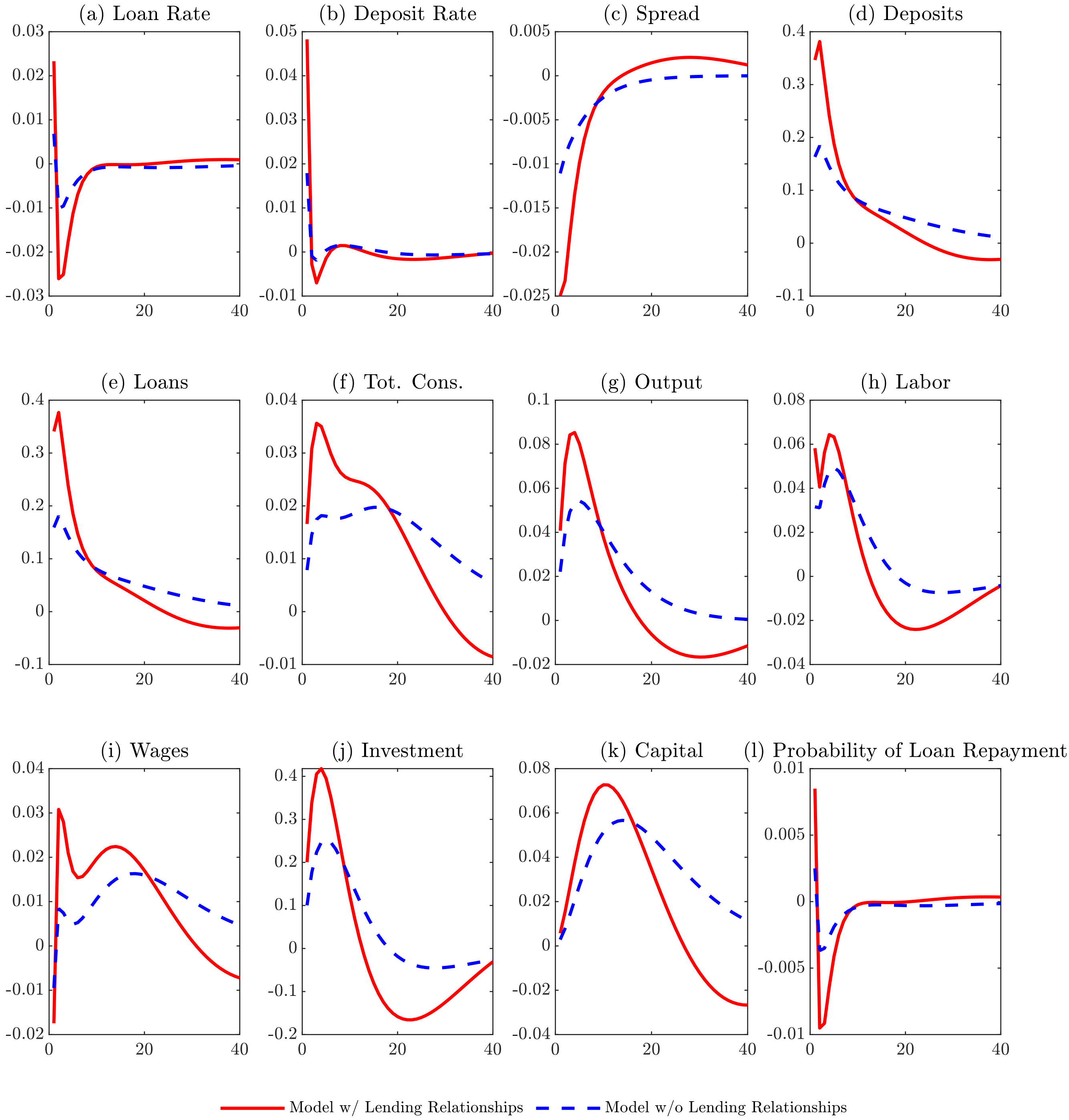
The parameter ϖ measures the elasticity of credit risk with respect to changes in LTV ratio. Using data from US mortgage loans originated between 1995 and 2008, excluding subprimes, Lam, Dunsky, and Kelly (2013) examine the impact of foreclosure and delinquency rates of higher LTV ratios at origination after controlling for borrower characteristics as well as housing and macroeconomic conditions. They report that foreclosure and delinquency rates tend to rise around one for one with the delinquency ratio, though this number differs between specifications. Von Furstenberg (1969) reports a higher elasticity 'in excess of unity'. The value of this elasticity is therefore chosen to be 1.5, that is $\varpi = -1.5$. Estimates of the value for η, entrepreneur's desire to minimize collateral pledges relative to cost minimization motive, are scarce. Booth and Booth (2006) find that firms' collateral minimization concern is of limited importance and they tend to choose the least costly form of borrowing. They point out that firms' willingness to accept higher lending rates in order ro reduce collateral requirements is rather small and therefore the value of η is set at 0.05 – a small value.

Following Smets and Wouters (2007), persistnce of technology shock σ_A is set to 0.95 and its standard deviation to 0.0014 which is standard in the literature. For credit shocks, I follow Pesaran and Xu (2016) and set the volatility of credit shock σ_ψ to 0.011 and autocorrelation parameter of volatility shock ρ_ψ to 0.848. I call these values baseline. I later conduct experiments in which I increase volatility by 50% and reduce autocorrelation parameter by 20%. These experiments allow me to capture in a transparent fashion the effects of credit shocks.

4 Results

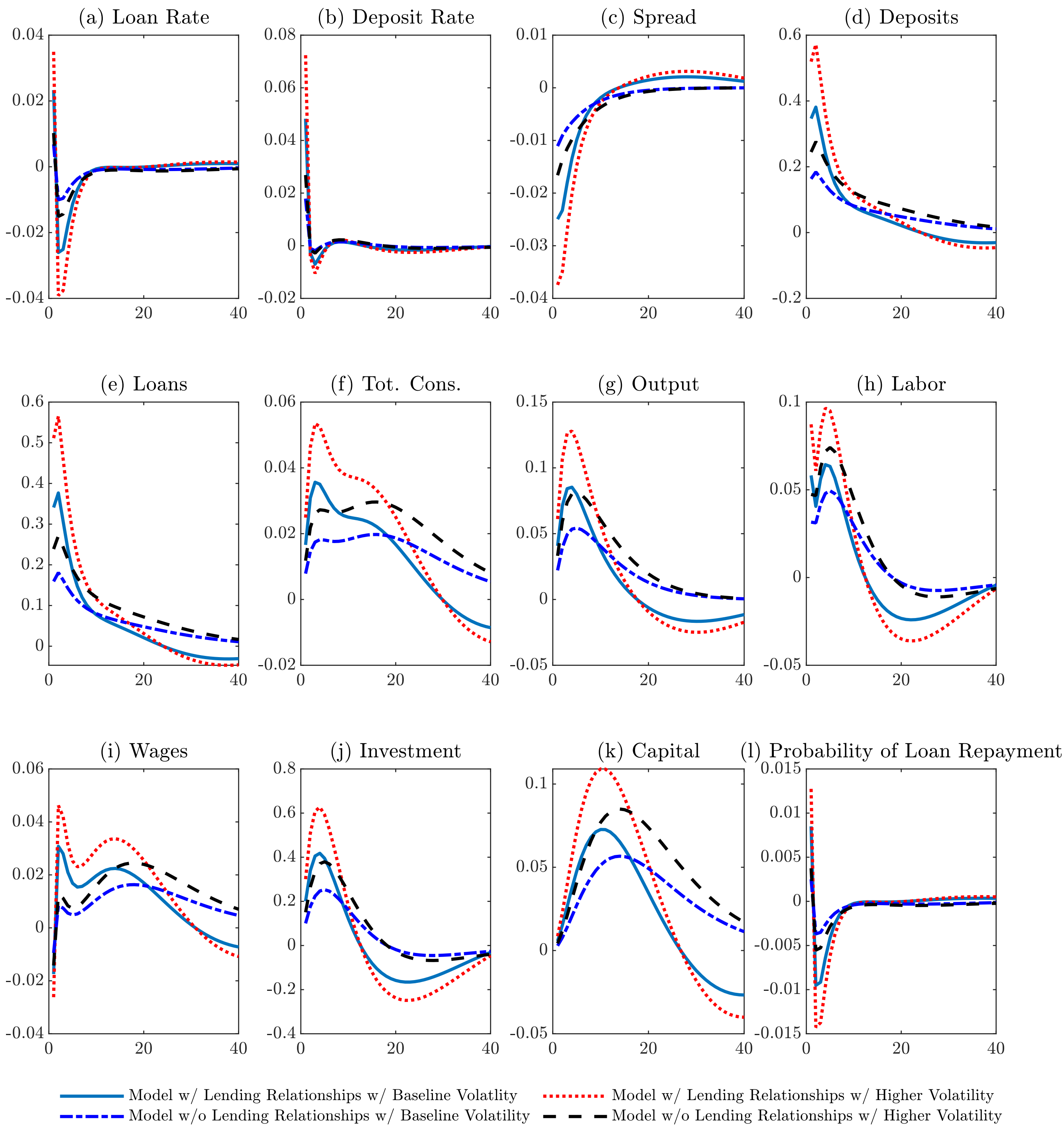
5 Conclusion

Figure 1: Impact of a Credit Shock



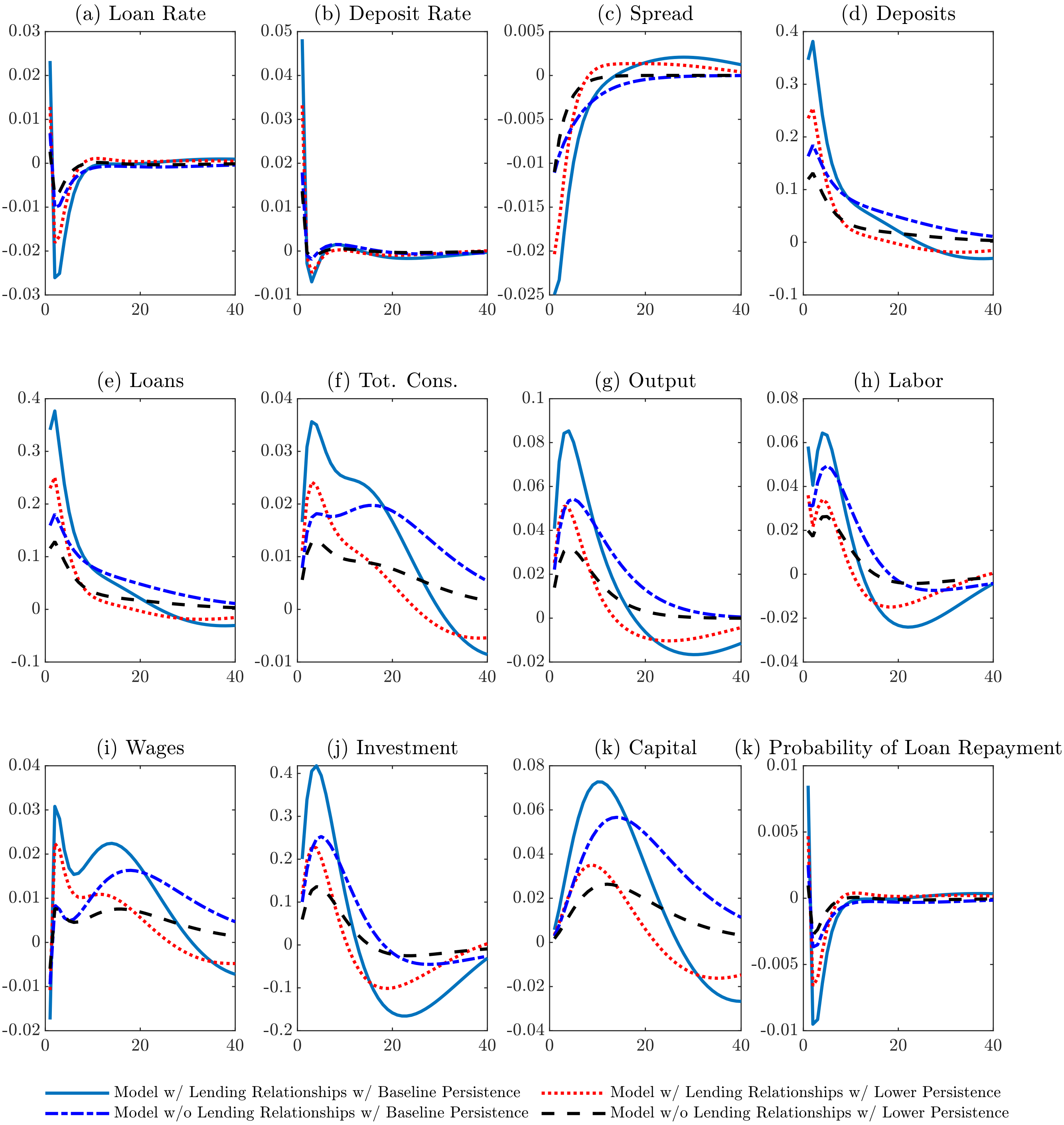
Note: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

Figure 2: Impact of a Credit Shock at Different Volatilities



Note: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

Figure 3: Impact of a Credit Shock at Different Peristence of Shock



Note: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

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Appendix (For Online Publication)

Credit Shocks, Endogenous Lending Standards and Macroeconomic Dynamics

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Contents

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A Derivation of FOCs

A.1 Households

The Lagrangian of patient households is

$$\mathscr{L}_t = \mathbb{E}_t \left\{ \textstyle\sum_{t=0}^{\infty} \left(\beta^P\right)^t \left[\begin{array}{c} \log\left(C_{i,t}^P - \gamma^P C_{i,t-1}^P\right) - \frac{N_{i,t}^{\nu}}{\nu} + \varsigma \log H_{i,t}^P \\ -\lambda_{i,t}^P \left[\begin{array}{c} C_{i,t}^P + Q_t^H \left(H_{i,t}^P - H_{i,t-1}^P\right) + \int_0^1 D_{ik,t} \mathrm{d}k \\ -W_t N_{i,t} - \int_0^1 \Pi_{ik,t} \mathrm{d}k - R_{t-1}^D \int_0^1 D_{ik,t-1} \mathrm{d}k \end{array} \right] \end{array} \right] \right\} \tag{A.1}$$

The problem yields the following first order conditions (here, I ignore all the i's denoting individual patient households):

$$\frac{\partial \mathscr{L}}{\partial C_t^P} : \frac{1}{C_t^P - \gamma^P C_{t-1}^P} - \beta^P \mathbb{E}_P \frac{\gamma^P}{C_{t+1}^P - \gamma^P C_t^P} = \lambda_t^P \tag{A.2}$$

$$\frac{\partial \mathscr{L}}{\partial D_t} : \beta^P \mathbb{E}_t \lambda_{t+1}^P = \frac{\lambda_t^P}{R_t^D} \tag{A.3}$$

$$\frac{\partial \mathscr{L}}{\partial H_t^P} : \frac{\varsigma_t}{H_t^P} + \beta^P \mathbb{E}_t \left(\lambda_{t+1}^P Q_{t+1}^H\right) = \lambda_t^P Q_t^H \tag{A.4}$$

$$\frac{\partial \mathscr{L}}{\partial N_t} : N_t^{\nu-1} = \lambda_t^P W_t \tag{A.5}$$

A.2 Entrepreneurs

Entrepreneur's optimization problem is identical to that in Ravn (2016). It also bears resemblance to entrepreneur's optimization problem in Sharma (2023b) in which banks compete only on interest rates and the economy does not feature a lending standard. Entrepreneur's optimization problem features two parts. The first part consists of choosing how much to borrow from each individual bank, $l_{jk,t}$ to minimize his total interest rate expenditure. This problem can be framed as

$$\min_{l_{jk,t}^E} \left[\int_0^1 R_{k,t}^L l_{jk,t}^E \mathrm{d}k + \eta \int_0^1 \frac{l_{jk,t}^E}{\theta_{k,t}} \mathrm{d}k \right] - \chi_t^E \left[x_{j,t}^E - \left(\int_0^1 \left(l_{jk,t}^E - \gamma^L s_{k,t-1}^E\right)^{\frac{\xi_t - 1}{\xi_t}} \mathrm{d}k \right)^{\frac{\xi_t}{\xi_t - 1}} \right] \tag{A.6}$$

This can be rewritten as

$$\min_{l^E_{jk,t}} \left[\int_0^1 \Upsilon_{k,t} l^E_{jk,t} \mathrm{d}k\right] - \chi^E_t \left[x^E_{j,t} - \left(\int_0^1 \left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)^{\frac{\xi_t-1}{\xi_t}} \mathrm{d}k\right)^{\frac{\xi_t}{\xi_t-1}}\right] \tag{A.7}$$

The first order condition for this problem is

$$R^L_{k,t} + \eta\frac{1}{\theta_{k,t}} = -\frac{\xi_t}{\xi_t-1}\chi^E_t \left(\int_0^1 \left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)^{\frac{\xi_t-1}{\xi_t}} \mathrm{d}k\right)^{\frac{1}{\xi_t-1}} \frac{\xi_t-1}{\xi_t}\left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)^{-\frac{1}{\xi_t}} \tag{A.8}$$

This can be rewritten as

$$\begin{aligned}
R^L_{k,t} + \eta\frac{1}{\theta_{k,t}} &= -\chi^E_t \left(\int_0^1 \left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)^{\frac{\xi_t-1}{\xi_t}} \mathrm{d}k\right)^{\frac{1}{\xi_t-1}} \left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)^{-\frac{1}{\xi_t}} \\
\left(R^L_{k,t} + \eta\frac{1}{\theta_{k,t}}\right)\left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right) &= -\chi^E_t \left(\int_0^1 \left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)^{\frac{\xi_t-1}{\xi_t}} \mathrm{d}k\right)^{\frac{1}{\xi_t-1}} \left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)^{\frac{\xi_t-1}{\xi_t}} \\
\int_0^1 \left(R^L_{k,t} + \eta\frac{1}{\theta_{k,t}}\right)\left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)\mathrm{d}k &= -\chi^E_t \left(\int_0^1 \left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)^{\frac{\xi_t-1}{\xi_t}} \mathrm{d}k\right)^{\frac{1}{\xi_t-1}} \int_0^1 \left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)^{\frac{\xi_t-1}{\xi_t}} \mathrm{d}k
\end{aligned}$$

$$\int_0^1 R^L_{k,t}\left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)\mathrm{d}k + \eta\int_0^1 \frac{1}{\theta_{k,t}}\left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)\mathrm{d}k = -\chi^E_t \left(\int_0^1 \left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)^{\frac{\xi_t-1}{\xi_t}} \mathrm{d}k\right)^{\frac{\xi_t}{\xi_t-1}} \tag{A.9}$$

Now, using $\left(\int_0^1 \left(l_{jk,t} - \gamma^L_t s_{k,t-1}\right)^{\frac{\xi_t-1}{\xi_t}} \mathrm{d}k\right)^{\frac{\xi_t}{\xi_t-1}} = x_{j,t}$, the previous equation can be written as

$$x_{j,t} = -\frac{1}{\chi^E_t}\left[\int_0^1 R^L_{k,t}\left(l_{jk,t} - \gamma^L_t s_{k,t-1}\right)\mathrm{d}k\right] \qquad \ddagger$$

Define the aggregate lending rate as $R^L_t \equiv \left[\int_0^1 \left(R^L_{k,t}\right)^{1-\xi_t}\right]^{\frac{1}{1-\xi_t}}$ and note that at the optimum, the following condition must hold

$$\frac{1}{\theta_t}x^E_{j,t} = \int_0^1 \frac{1}{\theta_{k,t}}\left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)\mathrm{d}k$$

Now, ‡ can be rewritten as

$$\begin{aligned}
x^E_{j,t} &= -\frac{1}{\chi^E_t}\left[R^L_t x^E_{j,t} + \eta\frac{1}{\theta_t}x^E_{j,t}\right] \\
-\chi^E_t &= R^L_t + \eta\frac{1}{\theta_t}
\end{aligned}$$

Inserting this in first order condition (A.9)

$$
\begin{aligned}
R^L_{k,t} + \eta\frac{1}{\theta_{k,t}} &= -\frac{\xi_t}{\xi_t - 1}\chi^E_t \left(\int_0^1 \left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)^{\frac{\xi_t-1}{\xi_t}} \mathrm{d}k\right)^{\frac{1}{\xi_t-1}} \frac{\xi_t - 1}{\xi_t} \left(l^E_{j,t} - \gamma^L s^E_{k,t-1}\right)^{-\frac{1}{\xi_t}} \\
R^L_{k,t} + \eta\frac{1}{\theta_{k,t}} &= \left(R^L_t + \eta\frac{1}{\theta_t}\right) \left(\int_0^1 \left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right) \mathrm{d}k\right)^{\frac{1}{\xi_t-1}} \left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)^{-\frac{1}{\xi_t}} \\
R^L_{k,t} + \eta\frac{1}{\theta_{k,t}} &= \left(R^L_{k,t} + \eta\frac{1}{\theta_t}\right) \left(x^E_t\right)^{\frac{1}{\xi_t}} \left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)^{-\frac{1}{\xi_t}} \\
\left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)^{\frac{1}{\xi_t}} &= \left(x^E_t\right)^{\frac{1}{\xi_t}} \frac{R^L_t + \eta\frac{1}{\theta_t}}{R^L_{k,t} + \eta\frac{1}{\theta_{k,t}}} \\
l^E_{jk,t} &= \left(\frac{R^L_t + \eta\frac{1}{\theta_t}}{R^L_{k,t} + \eta\frac{1}{\theta_{k,t}}}\right)^{\xi_t} x^E_t + \gamma^L s^E_{k,t-1} \\
l^E_{jk,t} &= \left(\frac{\Upsilon_t}{\Upsilon_{k,t}}\right)^{\xi_t} x^E_t + \gamma^L s^E_{k,t-1} \\
l^E_{jk,t} &= \left(\frac{\Upsilon_{k,t}}{\Upsilon_t}\right)^{-\xi_t} x^E_t + \gamma^L s^E_{k,t-1}
\end{aligned}
$$

When η is high, the entrepreneur attaches higher importance to collateral minimization motive. As a result, LTV ratios become more important for determination of demand for loans from each bank.

$$
\lim_{\eta\to 0}\left(\frac{\Upsilon_{k,t}}{\Upsilon}\right)^{-\xi_t} = \lim_{\eta\to 0}\left(\frac{R^L_{k,t} + \eta\frac{1}{\theta_{k,t}}}{R^L_t + \eta\frac{1}{\theta_t}}\right)^{-\xi_t} = \left(\frac{R^L_{k,t}}{R^L_t}\right)^{-\xi_t}
$$

The second part of entrepreneur's optimization problem can be written as

$$
\mathscr{L}_t = \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \left(\beta^E\right)^t \left[\begin{array}{c}
\log\left(C^E_{j,t} - \gamma^E C^E_{j,t-1}\right) \\
-\lambda^E_{j,t} \left[\begin{array}{c} C^E_{j,t} + R^L_{k,t-1}\int_0^1 l_{jk,t-1}\mathrm{d}k - Y_{j,t} + W_t N_{j,t} + I_{j,t} \\ +Q^H_t\left(H^E_{j,t} - H^E_{j,t-1}\right) - x_{j,t} - \Phi^E_t - \Psi^E_t \end{array} \right] \\
-\mu^E_{j,t}\left[R^L_{k,t}\int_0^1 l_{jk,t}\mathrm{d}k - \theta\mathbb{E}_t\left(Q^H_{t+1}H^E_{j,t} + Q^K_{t+1}K_{j,t}\right)\right] \\
-\kappa^E_{j,t}\left[K_{j,t} - (1-\delta)K_{j,t-1} - \left\{1 - \frac{\Omega}{2}\left(\frac{I_{j,t}}{I_{j,t-1}} - 1\right)^2\right\} I_{j,t}\right] \\
-\epsilon^E_{j,t}\left[x_{j,t} - \left\{\int_0^1 \left(l_{jk,t} - \gamma^L_t s_{k,t-1}\right)^{\frac{\xi_t-1}{\xi_t}} \mathrm{d}k\right\}^{\frac{\xi_t}{\xi_t-1}}\right]
\end{array} \right] \right\} \tag{A.10}
$$

where $Y_{j,t} = A_t \left(N_{j,t}\right)^{1-\alpha} \left\{ \left(H^E_{j,t-1}\right)^{\phi} \left(K_{j,t-1}\right)^{1-\phi} \right\}^{\alpha}$ may be inserted for $Y_{j,t}$ in the budget constraint. Solving entrepreneur's optmization problem, the first order conditions are (I ignore all j's here):

$$\frac{\partial \mathscr{L}}{\partial C^E_t} : \frac{1}{C^E_t - \gamma^E C^E_{t-1}} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C^E_{t+1} - \gamma^E C^E_t} = \lambda^E_t \tag{A.11}$$

$$\frac{\partial \mathscr{L}}{\partial x^E_t} : \lambda^E_t = \epsilon^E_t \tag{A.12}$$

$$\frac{\partial \mathscr{L}}{\partial l^E_t} : \epsilon^E_t = \beta^E \mathbb{E}_t \lambda^E_{t+1} R^L_t + \mu^E_t R^L_t \tag{A.13}$$

$$\frac{\partial \mathscr{L}}{\partial N_t} : W_t = (1-\alpha) \frac{Y_t}{N_t} \tag{A.14}$$

$$\frac{\partial \mathscr{L}}{\partial H^E_t} : \lambda^E_t Q^H_t = \beta^E \mathbb{E}_t \left\{ \lambda^E_{t+1} \left(Q^H_{t+1} + \alpha\phi \frac{Y_{t+1}}{H^E_t} \right) \right\} + \mu^E_t \theta \mathbb{E}_t Q^H_{t+1} \tag{A.15}$$

$$\frac{\partial \mathscr{L}}{\partial K_t} : \kappa^E_t = \alpha (1-\phi) \beta^E \mathbb{E}_t \left(\frac{\lambda^E_{t+1} Y_{t+1}}{K_t} \right) + \beta^E (1-\delta) \mathbb{E}_t \kappa^E_{t+1} + \mu^E_t \theta \mathbb{E}_t Q^K_{t+1} \tag{A.16}$$

$$\frac{\partial \mathscr{L}}{\partial I_t} : \lambda^E_t = \kappa^E_t \left\{ 1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Omega \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) \right\} + \beta^E \Omega \mathbb{E}_t \left\{ \kappa^E_{t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \right\} \tag{A.17}$$

Using $\lambda^E_t = \epsilon^E_t$ from (A.12), (A.13) becomes

$$\beta^E \mathbb{E}_t \left(\lambda^E_{t+1} R^L_t \right) + \mu^E_t R^L_t = \lambda^E_t \tag{A.18}$$

A.3 Banks

The problem of banks is to choose their lending rate, LTV ratio and the total amount of lending. The bank considers deep habits in loan demand. The bank solves the following problem

$$\begin{aligned} \max_{L_{k,t},\theta_{k,t},R^L_{k,t}} \Pi_t = {} & \left[\Xi + \varpi\left(\theta_{k,t} - \theta\right)\right] R^L_{k,t-1} L_{k,t-1} + \left[1 - \Xi + \varpi\left(\theta_{k,t} - \theta\right)\right] \frac{L_{k,t-1}}{\int_0^1 L_{k,t-1} \mathrm{d}k} \tau \theta_{t-1} a_{t-1} \\ & - R^D_{t-1} L_{k,t-1} + \mu^B_t \left(\int_0^1 \left[\left(\frac{R^L_t + \eta \frac{1}{\theta_t}}{R^L_{k,t} + \eta \frac{1}{\theta_{k,t}}} \right)^{\xi} x_t + \gamma^L s_{k,t-1} \right] \mathrm{d}j - L_{k,t} \right) \end{aligned}$$

The first order condition for $L_{k,t}$ is

$$\mathbb{E}_t q_{t,t+1} p_{k,t} R^L_{k,t} + \mathbb{E}_t q_{t,t+1} \left(1 - p_{k,t}\right) \frac{\tau \theta_t a_t}{\int_0^1 L_{k,t} \mathrm{d}k} - \mathbb{E}_t q_{t,t+1} R^D_t + \gamma^L \left(1 - \rho_s\right) \mathbb{E}_t \left(q_{t,t+1} \mu^B_{t+1} \right) - \mu^B_t = 0 \tag{A.19}$$

$$\mu^B_t = \mathbb{E}_t q_{t,t+1} \left[p_{k,t} R^L_{k,t} + \left(1 - p_{k,t}\right) \frac{\tau \theta_t a_t}{\int_0^1 L_{k,t} \mathrm{d}k} - R^D_t + \gamma^L \left(1 - \rho_s\right) \mathbb{E}_t \mu^B_{t+1} \right] \tag{A.20}$$

The first order condition for $\theta_{k,t}$ is

$$\varpi \mathbb{E}_t q_{t,t+1} R^L_{k,t} L_{k,t} - \varpi \mathbb{E}_t q_{t,t+1} \frac{L_{k,t}}{\int_0^1 L_{k,t} \mathrm{d}k} \tau \theta_t a_t + \xi \mu^B_t \left(\frac{R^L_t + \eta \frac{1}{\theta_t}}{R^L_{k,t} + \eta \frac{1}{\theta_{k,t}}} \right)^{\xi-1} x_t \left(\frac{\eta \frac{1}{\theta^2_{k,t} \left(R^L_t + \eta \frac{1}{\theta_t} \right)}}{R^L_{k,t} + \eta \frac{1}{\theta_{k,t}}} \right)^2 = 0 \tag{A.21}$$

$$\xi \mu^B_t x_t \left(\frac{R^L_t + \eta \frac{1}{\theta_t}}{R^L_{k,t} + \eta \frac{1}{\theta_{k,t}}} \right)^{\xi-1} \frac{\eta \frac{1}{\theta^2_{k,t}} \left(R^L_t + \eta \frac{1}{\theta_t} \right)}{\left(R^L_{k,t} + \eta \frac{1}{\theta_{k,t}} \right)^2} = -\varpi \mathbb{E}_t q_{t,t+1} \left[R^L_{k,t} L_{k,t} - \frac{L_{k,t}}{\int_0^1 L_{k,t} \mathrm{d}k} \tau \theta_t a_t \right] \tag{A.22}$$

The first order condition for $R^L_{k,t}$ is

$$\mathbb{E}_t q_{t.t+1} p_{k,t} L_{k,t} + \xi \mu^B_t \left(\frac{R^L_t + \eta \frac{1}{\theta_t}}{R^L_{k,t} + \eta \frac{1}{\theta_t}} \right)^{\xi-1} x_t \left(\frac{-\left(R^L_t + \eta \frac{1}{\theta_t} \right)}{\left(R^L_{k,t} + \eta \frac{1}{\theta_{k,t}} \right)^2} \right) \tag{A.23}$$

$$\mathbb{E}_t q_{t,t+1} p_{k,t} L_{k,t} = \xi \mu^B_t x_t \left(\frac{R^L_t + \eta \frac{1}{\theta_t}}{R^L_{k,t} + \eta \frac{1}{\theta_t}} \right)^{\xi-1} \left(\frac{\left(R^L_t + \eta \frac{1}{\theta_t} \right)}{\left(R^L_{k,t} + \eta \frac{1}{\theta_{k,t}} \right)^2} \right) \tag{A.24}$$

In a symmetric equilibrium all banks have the same lending rate $R^L_{k,t} = R^L_t, \forall k$ and lend the same amount $L_{k,t} = L_t, \forall k$. Bank's first order conditions in this case can be rewritten as

$$\varrho^I_t = \mathbb{E}_t q_{t,t+1} \left[p_{k,t} R^L_{k,t} + (1 - p_{k,t}) \frac{\tau \theta_t a^I_t}{\int_0^1 L^I_{k,t} \mathrm{d}k} - R^D_t + \gamma^L (1 - \rho_s) \mathbb{E}_t \varrho^I_{t+1} \right] \tag{A.25}$$

$$\xi_t \varrho^E_t x^E_t \frac{\frac{\eta}{\theta}}{R^L_t \theta_t + \eta} = -\mathbb{E}_t q_{t,t+1} R^L_t L^E_t \tag{A.26}$$

$$\xi_t \varrho^E_t x^E_t \frac{\frac{\eta}{\theta}}{R^L_t \theta_t + \eta} = 0 \tag{A.27}$$

$$\tag{A.28}$$

where I have imposed $L_t = l_t$ in a symmetric equilibrium and that the collateral constraint is always binding (holds with equality at all times).

B List of Equations

B.1 Households

$$\frac{1}{C^P_t - \gamma^P C^P_{t-1}} - \beta^P \mathbb{E}_t \frac{\gamma^P}{C^P_{t+1} - \gamma^P C^P_t} = \lambda^P_t \tag{B.1}$$

$$\beta^P \mathbb{E}_t \lambda^P_{t+1} = \frac{\lambda^P_t}{R^D_t} \tag{B.2}$$

$$\frac{\varsigma}{H_t^P} + \beta^P \mathbb{E}_t \left(\lambda_{t+1}^P Q_{t+1}^H \right) = \lambda_t^P Q_t^H \tag{B.3}$$

$$N_t^{\nu-1} = \lambda_t^P W_t \tag{B.4}$$

B.2 Entrepreneurs

$$\frac{1}{C_t^E - \gamma^E C_{t-1}^E} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C_{t+1}^E - \gamma^E C_t^E} = \lambda_t^E \tag{B.5}$$

$$\beta^E \mathbb{E}_t \left(\lambda_{t+1}^E R_t^L \right) + \mu_t^E R_t^L = \lambda_t^E \tag{B.6}$$

$$W_t = (1-\alpha) \frac{Y_t}{N_t} \tag{B.7}$$

$$\lambda_t^E Q_t^H = \beta^E \mathbb{E}_t \left\{ \lambda_{t+1}^E \left(Q_{t+1}^H + \alpha \phi \frac{Y_{t+1}}{H_t^E} \right) \right\} + \mu_t^E \theta \mathbb{E}_t Q_{t+1}^H \tag{B.8}$$

$$\kappa_t^E = \alpha (1-\phi) \beta^E \mathbb{E}_t \left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t} \right) + \beta^E (1-\delta) \mathbb{E}_t \kappa_{t+1}^E + \mu_t^E \theta \mathbb{E}_t Q_{t+1}^K \tag{B.9}$$

$$\lambda_t^E = \kappa_t^E \left\{ 1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Omega \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) \right\} + \beta^E \Omega \mathbb{E}_t \left\{ \kappa_{t+1}^E \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \right\} \tag{B.10}$$

$$s_t = \rho_s s_{t-1} + (1-\rho_s) l_t \tag{B.11}$$

$$x_t = \left(l_t - \gamma_t^L s_{t-1} \right) \tag{B.12}$$

$$L_t = l_t \tag{B.13}$$

$$C_t^E + R_{t-1}^L l_{t-1} = Y_t - W_t N_t - I_t - Q_t \left(H_t^E - H_{t-1}^E \right) + x_t + \Phi_t^E + \Psi_t^E \tag{B.14}$$

$$l_t = \frac{\theta a_t}{R_t^L} \tag{B.15}$$

$$a_t = \mathbb{E}_t \left(Q_{t+1}^H H_t^E + Q_{t+1}^K K_t \right) \tag{B.16}$$

$$\kappa_t^E = \lambda_t^E Q_t^K \tag{B.17}$$

B.3 Banks

$$\Pi_{k,t} = R_{k,t-1}^L L_{k,t-1} + \int_0^1 D_{ik,t} \mathrm{d}i - L_{k,t} - R_{t-1}^D \int_0^1 D_{ik,t-1} \mathrm{d}i \tag{B.18}$$

$$L_t = \psi_t D_t \tag{B.19}$$

$$q_{t,t+1} = \beta^P \mathbb{E}_t \frac{\lambda_{t+1}^P}{\lambda_t^P} \tag{B.20}$$

$$p_t = \Xi + \varpi (\theta_t - \theta) \tag{B.21}$$

$$\mu_t^B = \mathbb{E}_t q_{t,t+1} \left\{ p_t R_t^L + (1-p_t)\, \tau \frac{\theta_t a_t}{L_t} - R_t^D + \gamma^L (1-\rho_s)\, \mathbb{E}_t \mu_{t+1}^B \right\} \tag{B.22}$$

$$\xi \mu_t^B x_t \frac{\frac{\eta}{\theta_t}}{R_t^L + \eta} = -\varpi \mathbb{E}_t q_{t,t+1} \left(R_t^L L_t - \tau \theta_t a_t \right) \tag{B.23}$$

$$\xi \mu_t^B x_t \frac{\theta_t}{\theta_t R_t^L + \eta} = \mathbb{E}_t q_{t,t+1} p_t L_t \tag{B.24}$$

B.4 Market Clearing and Resource Constraints

$$C_t^P + C_t^E + I_t = Y_t \tag{B.25}$$

$$H_t^P + H_t^E = H \tag{B.26}$$

$$Y_t = A_t (N_t)^{1-\alpha} \left\{ \left(H_{t-1}^E \right)^{\phi} (K_{t-1})^{1-\phi} \right\}^{\alpha} \tag{B.27}$$

$$K_t = (1-\delta) K_{t-1} + \left\{ 1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right\} I_t \tag{B.28}$$

C Steady State Conditions

All $i's$, $j's$ and $k's$ denoting individual household, entrepreneur and bank respectively are ignored. From household's FOC with respect to consumption (B.1) and labor (B.4), I have

$$\frac{1-\beta^P \gamma^P}{(1-\gamma^P)\, C^P} = \lambda^P \tag{C.1}$$

and

$$N^{\nu-1} = \lambda^P W \tag{C.2}$$

respectively. Household's FOC with respect to deposit (B.2) yields the steady-state gross interest rate

$$R^D = \frac{1}{\beta^P} \tag{C.3}$$

underscoring that the time preference of the most patient agent determines the steady-state rate of interest. From (B.3), I obtain

$$\begin{aligned} \frac{\varsigma}{H^P} + \beta^P \lambda^P Q^H &= \lambda^P Q^H \\ \Rightarrow Q^H H^P &= \frac{\varsigma}{\lambda^P (1-\beta^P)} \\ \Rightarrow H^P &= \frac{\varsigma}{Q^H \lambda^P (1-\beta^P)} \end{aligned} \tag{C.4}$$

I next turn to entrepreneurs. Their consumption FOC (B.5) yields

$$\frac{1-\beta^E\gamma^E}{(1-\gamma^E)\,C^E}=\lambda^E \tag{C.5}$$

Entrepreneur's FOC with respect to loans (B.6) gives

$$\begin{aligned}\beta^E\lambda^E R^L+\mu^E R^L &= \lambda^E\\ \Rightarrow \mu^E &= \frac{\lambda^E\left(1-\beta^E R^L\right)}{R^L}\end{aligned} \tag{C.6}$$

The borrowing constraint for entrepreneurs binds only if μ^E is positive. This implies that β^E must be less than R^L. In the baseline calibration, β^E is set to 0.95 whereas the steady state value of R^L is 1.0219 which implies that β^E must be less than 0.9786 which is indeed the case. The production function is

$$Y=A\,(N)^{1-\alpha}\left[\left(H^E\right)^{\phi}(K)^{1-\phi}\right]^{\alpha} \tag{C.7}$$

From firm's labor choice for patient househods (B.7),

$$W=(1-\alpha)\,\frac{Y}{N} \tag{C.8}$$

From entrepreneur's FOC with respect to housing (B.8), I have

$$\begin{aligned}\lambda^E Q^H &= \beta^E\lambda^E\left(Q^H+\alpha\phi\frac{Y}{H^E}\right)+\mu^E\theta Q^H\\ \Rightarrow \frac{Q^H H^E}{Y} &= \frac{\beta^E\alpha\phi R^L}{\left(1-\beta^E\right)R^L-\theta\left(1-\beta^E R^L\right)}\end{aligned} \tag{C.9}$$

From aggregate law of motion for capital (B.28)

$$\begin{aligned}K &= (1-\delta)\,K+\left[1-\frac{\Omega}{2}\left(\frac{I}{I}-1\right)\right]I\\ &\Rightarrow I=\delta K\end{aligned} \tag{C.10}$$

I have the following steady-state resource constraints

$$Y=C^P+C^E+I \tag{C.11}$$

$$H=H^P+H^E \tag{C.12}$$

$$L=\psi D \tag{C.13}$$

Also, I have the following steady-state version of agents' budget constraints (one of them is redundant because of Walras' Law)

$$C^P = WN - (R^D - 1)D + \Pi \tag{C.14}$$

$$C^E = Y - R^L l - WN - I - x \tag{C.15}$$

The steady state, therefore, is characterized by the vector

$$\left[Y, C^P, C^E, I, H^P, H^E, K, N, W, L, D, Q^H, Q^K, R^D, R^L, \lambda^P, \lambda^E, \mu^E\right]$$

From entrepreneur's optimal choice of capital (B.9), I have

$$\begin{aligned}
\kappa_t^E &= \alpha\left(1-\alpha\right)\beta^E \mathbb{E}_t\left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t}\right) + \beta^E\left(1-\delta\right)\mathbb{E}_t\kappa_{t+1}^E + \mu_t^E\theta_t\mathbb{E}_t Q_{t+1}^K \\
\Rightarrow \frac{\kappa^E}{\lambda^E}\left(1-\left(1-\delta\right)\beta^E\right) &= \alpha\left(1-\phi\right)\beta^E\frac{Y}{K} + \frac{\left(1-\beta^E R^L\right)}{R^L}\theta Q^K
\end{aligned} \tag{C.16}$$

Entrepreneur's optimal choice of investment (B.10) yields

$$\begin{aligned}
\lambda_t^E(j) = \kappa_t^E(j) & \left[1 - \frac{\Omega}{2}\left(\frac{I_t(j)}{I_t(j-1)} - 1\right)^2 - \Omega\frac{I_t(j)}{I_t(j-1)}\left(\frac{I_t(j)}{I_{t-1}(j)} - 1\right)\right] \\
& + \beta^E\Omega\mathbb{E}_t\left[\kappa_{t+1}^E(j)\left(\frac{I_{t+1}(j)}{I_t(j)}\right)^2\left(\frac{I_{t+1}(j)}{I_t(j)} - 1\right)\right] \\
\Rightarrow \lambda^E = \kappa^E &
\end{aligned} \tag{C.17}$$

Combining this with steady state version of

$$\kappa^E = \lambda^E Q^K \tag{C.18}$$

I obtain $Q^K = 1$ in the steady state. Plugging this into (C.16), I obtain the expression for capital-to-output ratio

$$\begin{aligned}
\frac{\kappa^E}{\lambda^E}\left(1-\left(1-\delta\right)\beta^E\right) &= \alpha\left(1-\phi\right)\beta^E\frac{Y}{K} + \frac{\left(1-\beta^E R^L\right)}{R^L}\theta Q^K \\
\Rightarrow \frac{K}{Y} &= \frac{\alpha\left(1-\phi\right)R^L\beta^E}{R^L\left(1-\left(1-\delta\right)\beta^E\right) - \theta\left(1-\beta^E R^L\right)}
\end{aligned} \tag{C.19}$$

Next, combining (B.15) and (B.16)

$$l = \frac{\theta}{R^L}\left(Q^H H^E + Q^K K\right) \tag{C.20}$$

Dividing by Y, the above expression becomes

$$
\begin{aligned}
\frac{l}{Y} &= \frac{\theta}{R^L}\left(\frac{Q^H H^E}{Y} + \frac{Q^K K}{Y}\right) \\
\Rightarrow \frac{l}{Y} &= \alpha\theta\beta^E\left[\frac{\phi}{R^L\left(1-\beta^E\right)-\theta\left(1-\beta^E R^L\right)} + \frac{(1-\phi)}{R^L\left(1-(1-\delta)\,\beta^E\right)-\theta\left(1-\beta^E R^L\right)}\right]
\end{aligned} \tag{C.21}
$$

From entrepreneur's budget constraint (B.14)

$$
C^E + R^L l = Y - WN - I + x + \Phi^E + \Psi^E \tag{C.22}
$$

Rewriting this in ratios to output

$$
\begin{aligned}
\frac{C^E}{Y} + \frac{R^L l^E}{Y} &= 1 - \frac{W^P N^P}{Y} - \frac{I}{Y} + \frac{x^E}{Y} + \frac{\Phi^E}{Y} + \frac{\Psi^E}{Y} \\
\Rightarrow \frac{C^E}{Y} &= \alpha - \delta\frac{K}{Y} + \left(1-\gamma^L - R^L\right)\frac{l^E}{Y} + \frac{\Phi^E}{Y} + \frac{\Psi^E}{Y}
\end{aligned} \tag{C.23}
$$

Further simplifying the expression

$$
\begin{aligned}
\frac{C^E}{Y} &= \alpha - \delta\frac{K}{Y} + \left(1-\gamma^L - R^L\right)\frac{l^E}{Y} + \frac{\gamma^L s^E}{Y} + \frac{(1-p)\left(R^L L^E - \tau\theta a^E\right)}{Y} \\
\Rightarrow \frac{C^E}{Y} &= \alpha - \delta\frac{K}{Y} + \left[1 - pR^L - (1-p)\tau R^L\right]\frac{l^E}{Y}
\end{aligned} \tag{C.24}
$$

Steady-state budget constraint of patient household, in ratio to output, reads

$$
\begin{aligned}
\frac{C^P}{Y} &= \frac{W^P N^P}{Y} + \left(R^D - 1\right)\frac{D}{Y} + \frac{\Pi}{Y} \\
&= (1-\alpha) + \frac{\left(R^D-1\right)\left(L^E + L^I\right)}{Y} + \frac{\left(pR^L - R^D\right)\left(L^E+L^I\right) + (1-p)\,\tau\theta\left(a^E + a^I\right)}{Y}
\end{aligned} \tag{C.25}
$$

Dividing the above two expressions by each other, I have

$$
\begin{aligned}
\frac{\frac{Q^H H^P}{Y}}{\frac{Q^H H^E}{Y}} &= \frac{\frac{\varsigma}{Y\lambda^P\left(1-\beta^P\right)}}{\frac{\beta^E\alpha\phi R^L}{\left(1-\beta^P\right)R^L - \theta\left(1-\beta^E R^L\right)}} \\
\Rightarrow \frac{H^P}{H^E} &= \frac{\varsigma}{Y\frac{1-\beta^P\gamma^P}{\left(1-\gamma^P\right)C^P}\left(1-\beta^P\right)}\,\frac{\left(1-\beta^E\right)R^L - \theta\left(1-\beta^E R^L\right)}{\beta^E\alpha\phi R^L} \\
\Rightarrow \frac{H^P}{H-H^P} &= \frac{\varsigma\left(1-\gamma^P\right)}{\left(1-\beta^P\right)\left(1-\beta^P\gamma^P\right)}\,\frac{\left(1-\beta^P\right)R^L - \theta\left(1-\beta^E R^L\right)}{\beta^E\alpha\phi R^L}\,\frac{C^P}{Y}
\end{aligned} \tag{C.26}
$$

From entrepreneur's stock of habits for loans (B.11)

$$s = \rho_s s + (1 - \rho_s)\, l$$

which leads to

$$s = l \tag{C.27}$$

Entrepreneur's effective demand for loans (B.12) gives

$$x = \left(l - \gamma^L s\right)$$

Using $s = l$, this can be written as

$$x = \left(1 - \gamma^L\right) l \tag{C.28}$$

Total loans of entrepreneurs (B.13)

$$L = l \tag{C.29}$$

From bank's balance sheet condition (B.19)

$$D = \psi L \tag{C.30}$$

Steady state version of stochastic discount factor (B.20) reads

$$q = \beta^P \tag{C.31}$$

Now using the previous result and $\frac{\theta a}{L} = R^L$

$$\varrho^E = \beta^P \left[p R^L + (1 - p)\, \tau R^L - R^D + \gamma^L (1 - \rho_s)\, \varrho^E \right]$$

which can be rewritten as

$$\varrho^E = \beta^P \frac{p R^L + (1 - p)\, \tau R^L - R^D}{1 - \beta^P \gamma^L (1 - \rho_s)} \tag{C.32}$$

From bank's second FOC

$$\xi \varrho^E \left(\frac{\frac{\eta}{\theta}}{\theta R^L + \eta} \right) x = -\varpi \beta^P \left(R^L L - \tau \theta a \right)$$

After subsituting the expression for x

$$\xi \varrho^E \left(\frac{\frac{\eta}{\theta}}{\theta R^L + \eta} \right) \left(1 - \gamma^L\right) l = -\varpi \beta^P \left(R^L l - \tau R^L l \right)$$

This finally simplifies to

$$\xi \varrho^E \left(\frac{\frac{\eta}{\theta}}{\theta R^L + \eta} \right) \left(1 - \gamma^L\right) = -\varpi \beta^P R^L \left(1 - \tau\right) \tag{C.33}$$

The final FOC of banks optimization problem reads

$$\xi \varrho^E \left(\frac{\theta}{\theta R^L + \eta} \right) x = \beta^P pL$$

Rewriting this equation

$$\begin{aligned}
\xi \varrho^E \left(\frac{\theta}{\theta R^L + \eta} \right) \left(1 - \gamma^L\right) &= \beta^P p \\
\Rightarrow \xi \varrho^E \left(1 - \gamma^L\right) \frac{\theta}{\theta R^L + \eta} &= \beta^P p \\
\Rightarrow \xi \varrho^E \left(1 - \gamma^L\right) \theta &= \beta^P p \left(\theta R^L + \eta\right) \\
\Rightarrow \theta \Big[\xi \varrho^E \left(1 - \gamma^L\right) - \beta^P p R^L \Big] &= \beta^P p \eta \\
\Rightarrow \theta &= \frac{\beta^P p \eta}{\xi \varrho^E \left(1 - \gamma^L\right) - \beta^P p R^L}
\end{aligned} \tag{C.34}$$

(C.32), (C.33) and (C.34) form a system of 3 equations in 3 unknowns: ϱ^E, θ and R^L. In order to solve this syetm of equations, I first insert for ϱ^E from (C.32) into (C.33) and (C.34). This gives the following system of equation

$$\begin{aligned}
\xi \left(1 - \gamma^L\right) \frac{pR^L + (1-p)\,\tau R^L - R^D}{1 - \beta^P \gamma^L \left(1 - \rho_s\right)} \frac{\eta}{\theta} &= -\varpi R^L \left(1 - \tau\right) \left(\theta R^L + \eta\right) \\
\theta &= \frac{\beta^P p \eta}{\xi \left(1 - \gamma^L\right) \beta^P \dfrac{pR^L + (1-p)\,\tau R^L - R^D}{1 - \beta^P \gamma^L \left(1 - \rho_s\right)} - \beta^P p R^L}
\end{aligned}$$

Plugging the value of θ from the second equation into the first, I obtain the value of R^L after which values of ϱ^E and θ follow directly. This procedure determines the value of R^L exclusively from bank's problem which allows it to be inserted into equations derived from entrepreneur's problem.

Steady state version of aggregate resource constraint (B.25) is

$$\begin{aligned}
C^P + C^E + I &= Y \\
\Rightarrow \frac{C^P}{Y} &= 1 - \frac{C^E}{Y} - \delta \frac{K}{Y}
\end{aligned} \tag{C.35}$$

From steady state value of (B.21)

$$p = \Xi \tag{C.36}$$

Combining (C.1), (C.2) and (C.8) gives steady-state equilibrium condition for households

$$\begin{aligned} N^{\nu-1} &= \lambda^P W \\ \Rightarrow N^{\nu-1} &= \frac{1-\beta^P\gamma^P}{\left(1-\gamma^P\right)C^P}\left(1-\alpha\right)\frac{Y}{N} \\ \Rightarrow N &= \left[\frac{\left(1-\beta^P\gamma^P\right)\left(1-\alpha\right)}{\left(1-\gamma^P\right)}\left(\frac{C^P}{Y}\right)^{-1}\right]^{\frac{1}{\nu}} \end{aligned} \tag{C.37}$$

From (B.27), steady state output is

$$\begin{aligned} Y &= A\left(N\right)^{1-\alpha}\left[\left(H^E\right)^{\phi}\left(K\right)^{1-\phi}\right]^{\alpha} \\ Y^{1-\alpha} &= A\left(N\right)^{1-\alpha}\left[\left(\frac{H^E}{Y}\right)^{\phi}\left(\frac{K}{Y}\right)^{1-\phi}\right]^{\alpha} \\ Y^{1-\alpha} &= A\left(N\right)^{1-\alpha}\left[\left(\frac{H^E}{Y}\right)^{\phi}\left(\frac{\alpha\left(1-\phi\right)R^L\beta^E}{R^L\left(1-\left(1-\delta\right)\beta^E\right)-\theta\left(1-\beta^E R^L\right)}\right)^{1-\phi}\right]^{\alpha} \end{aligned} \tag{C.38}$$

From Equation (C.4)

$$Q^H = \frac{\varsigma}{H^P\lambda^P\left(1-\beta^P\right)} \tag{C.39}$$

D System of Loglinear Equations

The system of equations log-linearized around their steady state is as below:

D.1 Optimality Conditions of Households

Equations (B.1), (B.2) and (B.4) become

$$\beta^P\gamma^P\mathbb{E}_t\widehat{C}^P_{t+1} - \left(1+\left(\gamma^P\right)^2\beta^P\right)\widehat{C}^P_t + \gamma^P\widehat{C}^P_{t-1} = \left(1-\beta^P\gamma^P\right)\left(1-\gamma^P\right)\widehat{\lambda}^P \tag{D.1}$$

$$\mathbb{E}_t\widehat{\lambda}^P_{t+1} = \widehat{\lambda}^P_t - \widehat{R}^D_t \tag{D.2}$$

$$\left(\nu-1\right)\widehat{N}_t = \widehat{\lambda}^P_t + \widehat{W}_t \tag{D.3}$$

Log-linearization of (B.3) yields

$$\beta^P\mathbb{E}_t\left[\widehat{\lambda}^P_{t+1} + \widehat{Q}^H_{t+1} + \widehat{H}^P_t\right] = \widehat{\lambda}^P_t + \widehat{Q}^H_t + \widehat{H}^P_t \tag{D.4}$$

D.2 Optimality Conditions of Entrepreneurs

From (B.5) and (B.6), I have

$$\beta^E \gamma^E \mathbb{E}_t \widehat{C}^E_{t+1} - \left(1 + \left(\gamma^E\right)^2 \beta^E\right) \widehat{C}^E_t + \gamma^E \widehat{C}^E_{t-1} = \left(1 - \beta^E \gamma^E\right) \left(1 - \gamma^E\right) \widehat{\lambda}^E_t \tag{D.5}$$

and

$$\widehat{\lambda}^E_t = \widehat{R}^L_t + \beta^E R^L \mathbb{E}_t \widehat{\lambda}^E_{t+1} + \left(1 - \beta^E R^L\right) \widehat{\mu}^E_t \tag{D.6}$$

Equation (B.7) yields

$$\widehat{W}_t = \widehat{Y}_t - \widehat{N}_t \tag{D.7}$$

From (B.8), I derive

$$\begin{aligned} \left(\widehat{\lambda}^E_t + \widehat{Q}^H_t\right) = &\beta^E \mathbb{E}_t \left(\widehat{\lambda}^E_{t+1} + \widehat{Q}^H_{t+1}\right) + \left(\frac{1}{R^L} - \beta^E\right) \theta \mathbb{E}_t \left(\widehat{\mu}^E_t + \widehat{\theta}_t + \widehat{Q}^H_{t+1}\right) \\ &+ \left[\left(1 - \beta^E\right) - \theta \left(\frac{1}{R^L} - \beta^E\right)\right] \mathbb{E}_t \left[\widehat{\lambda}^E_{t+1} + \widehat{Y}_{t+1} - \widehat{H}^E_t\right] \end{aligned} \tag{D.8}$$

Equation (B.9) becomes

$$\begin{aligned} \widehat{Q}^K_t = &\left[1 - \beta^E (1 - \delta) - \theta \left(\frac{1}{R^L} - \beta^E\right)\right] \mathbb{E}_t \left[\widehat{\lambda}^E_{t+1} - \lambda^E_t + \widehat{Y}_{t+1} - K_t\right] \\ &+ \beta^E (1 - \delta) \mathbb{E}_t \left(\widehat{Q}^K_{t+1} + \widehat{\lambda}^E_{t+1} - \widehat{\lambda}^E_t\right) + \left(1 - \beta^E R^L\right) \frac{1}{R^L} \theta \mathbb{E}_t \left[\widehat{\mu}^E_t - \widehat{\lambda}^E_t + \widehat{\theta}_t + \widehat{Q}^K_{t+1}\right] \end{aligned} \tag{D.9}$$

Equation (B.10) is approximated as

$$\widehat{Q}^K_t = \left(1 + \beta^E\right) \Omega \widehat{I}_t - \beta^E \Omega \mathbb{E}_t \widehat{I}_{t+1} - \Omega \widehat{I}_{t-1} \tag{D.10}$$

From (B.11) and (B.12), I get

$$\widehat{s}_t = \rho_s \widehat{s}_{t-1} + (1 - \rho_s) \widehat{l}_t \tag{D.11}$$

and

$$\widehat{x}_t = \frac{\widehat{l}_t}{1 - \gamma^L} - \frac{\gamma^L \widehat{s}_{t-1}}{1 - \gamma^L} \tag{D.12}$$

Entrepreneurs' budget contraint (B.14) becomes

$$
\begin{aligned}
C^E \widehat{C}_t^E + R^L l \left(\widehat{R}_{t-1}^L + \widehat{l}_{t-1}\right) = Y \widehat{Y}_t - WN \left(\widehat{W}_t + \widehat{N}_t\right) - I \widehat{I}_t - Q^H H^E \left(\widehat{H}_t^E - \widehat{H}_{t-1}^E\right) + x \widehat{x}_t \\
+ \gamma^L s \widehat{s}_{t-1} + R^L L \left(\widehat{R}_{t-1}^L + \widehat{L}_{t-1}\right) - \tau a \widehat{a}_{t-1} \\
- p R^L L \left(\widehat{p}_{t-1} + \widehat{R}_{t-1}^L + \widehat{L}_{t-1}\right) + \tau p a \left(\widehat{p}_{t-1} + \widehat{a}_{t-1}\right)
\end{aligned}
\tag{D.13}
$$

The borrowing constraint (B.15) yields

$$
\widehat{l}_t = \widehat{\theta}_t + \widehat{a}_t - \widehat{R}_t^L \tag{D.14}
$$

The definition of entrepreneurs' total assets (B.16) gives

$$
\widehat{a}_t = \frac{Q^H H^E}{Q^H H^E + Q^K K} \mathbb{E}_t \left(\widehat{Q}_{t+1}^H + \widehat{H}_t^E\right) + \frac{Q^K K}{Q^H H^E + Q^K K} \mathbb{E}_t \left(\widehat{Q}_{t+1}^K + \widehat{K}_t\right) \tag{D.15}
$$

Linearized version of (B.17) is

$$
\widehat{\kappa}_t^E = \widehat{\lambda}_t^E + \widehat{Q}_t^K \tag{D.16}
$$

D.3 Optimality Conditions of Banks

From (B.22), I obtain

$$
\begin{aligned}
\frac{\varrho^E}{\beta^P} \widehat{\varrho}_t^E - \varrho^E \gamma^L \left(1-\rho_s\right) \mathbb{E}_t \varrho_{t+1}^E = \left[p R^L + (1-p) \tau R^L - R^D + \varrho^E \gamma^L \left(1-\rho_s\right)\right] \mathbb{E}_t \widehat{q}_{t,t+1} \\
+ p R^L \left(\widehat{p}_t + \widehat{R}_t^L\right) - R^D \widehat{R}_t^D + (1-p) \tau R^L \widehat{R}_t^L - p \tau R^L \widehat{p}_t
\end{aligned}
\tag{D.17}
$$

Equation (B.23) becomes

$$
\begin{aligned}
\frac{\eta \xi \varrho^E x}{\theta} \left(\widehat{\varrho}_t^E + \widehat{x}_t - \widehat{\theta}_t\right) = -\varpi \beta^P \left(R^L\right)^2 L \theta \left(2 \widehat{R}_t^L + \widehat{L}_t + \widehat{\theta}_t + \mathbb{E}_t \widehat{q}_{t,t+1}\right) \\
- \eta \varpi \beta^P R^L L \left(\widehat{R}_t^L + \widehat{L}_t + \mathbb{E}_t \widehat{q}_{t,t+1}\right) \\
+ \varpi \tau \beta^P a \theta^2 R^L \left(\widehat{a}_t + 2 \widehat{\theta}_t + \widehat{R}_t^L + \mathbb{E}_t \widehat{q}_{t,t+1}\right) \\
+ \eta \varpi \tau \beta^P \theta a \left(\widehat{a}_t + \widehat{\theta}_t + \mathbb{E}_t \widehat{q}_{t,t+1}\right)
\end{aligned}
\tag{D.18}
$$

From (B.24), I get

$$
\begin{aligned}
\xi \varrho^E x \theta \left(\widehat{\varrho}_t^E + \widehat{x}_t + \widehat{\theta}_t\right) = \theta \beta^P R^L p L \left(\widehat{\theta}_t + \widehat{R}_t^L + \widehat{p}_t + \widehat{L}_t + \mathbb{E}_t \widehat{q}_{t,t+1}\right) \\
+ \eta \beta^P p L \left(\widehat{p}_t + \widehat{L}_t + \mathbb{E}_t \widehat{q}_{t,t+1}\right)
\end{aligned}
\tag{D.19}
$$

Linearized versions of (B.20) and (B.21) are

$$\widehat{q}_{t,t+1} = \widehat{\lambda}_{t+1}^{P} - \widehat{\lambda}_{t}^{P} \tag{D.20}$$

and

$$p\widehat{p}_t + \zeta\widehat{\zeta}_t = \varpi\theta\widehat{\theta}_t \tag{D.21}$$

Equation (B.19) gives

$$L\widehat{L}_t = \psi\widehat{\psi}_t + D\widehat{D}_t \tag{D.22}$$

D.4 Market Clearing and Resource Constraints

Equations (B.25) and (B.26) yield

$$\widehat{Y}_t = \frac{C^P}{C}\widehat{C}_t^P + \frac{C^E}{Y}\widehat{C}_t^E + \frac{I}{Y}\widehat{I}_t \tag{D.23}$$

and

$$H^P\widehat{H}_t^P + H^E\widehat{H}_t^E = 0 \tag{D.24}$$

From (B.27) we have

$$\widehat{Y}_t = \widehat{A}_t + (1-\alpha)\,\widehat{N}_t + \alpha\phi\widehat{H}_{t-1}^E + \alpha\,(1-\phi)\,\widehat{K}_{t-1} \tag{D.25}$$

Equation (B.28) gives

$$\widehat{K}_t = (1-\delta)\,\widehat{K}_{t-1} + \delta\widehat{I}_t \tag{D.26}$$

E Market Clearing

The derivation of market clearing conditon is identical to Ravn (2016) and I include it here for the sake of completeness. As mentioned in the main text, two types of transfers Ψ_t and Φ_t to entrepreneurs are needed to ensure all markets clear. This section demonstrates this and shows the derivation of the expression for Ψ_t. Let's start by adding together the budget constraints of households and entrepreneurs. We sum over both households and entrepreneurs, respectively:

$$\begin{aligned}
&\int_0^1 \left(C_{i,t}^P + Q_t^H\left(H_{i,t}^P - H_{i,t-1}^P\right) + \int_0^1 D_{ik,t}\mathrm{d}k\right)\mathrm{d}i + \int_0^1 \left(C_{j,t}^E + R_{t-1}^L\int_0^1 l_{jk,t-1}\mathrm{d}k\right)\mathrm{d}j \\
&= \int_0^1 \left(W_tN_{i,t} + \int_0^1 \Pi_{ik,t}\mathrm{d}k + R_{t-1}^D\int_0^1 D_{ik,t-1}\mathrm{d}k\right)\mathrm{d}i \\
&+ \int_0^1 \left(Y_{j,t} - W_tN_{j,t} - I_{j,t} - Q_t^H\left(H_{j,t}^E - H_{j,t-1}^E\right) + x_{j,t} + \Phi_t + \Psi_t\right)\mathrm{d}j
\end{aligned}$$

After doing the outer integral, I obtain:

$$
\begin{aligned}
&C_t^P + Q_t^H\left(H_t^P - H_{t-1}^P\right) + \int_0^1\int_0^1 D_{ik,t}\mathrm{d}i\mathrm{d}k + C_t^E + R_{t-1}^L\int_0^1\int_0^1 l_{jk,t-1}\mathrm{d}k\mathrm{d}j \\
&= W_tN_t + \int_0^1\int_0^1 \Pi_{ik,t}\mathrm{d}k\mathrm{d}i + R_{t-1}^D\int_0^1\int_0^1 D_{ik,t-1}\mathrm{d}k\mathrm{d}i \\
&+ Y_t - W_tN_t - I_t - Q_t^H\left(H_t^E - H_{t-1}^E\right) + \int_0^1 x_{j,t}\mathrm{d}j + \int_0^1 \Phi_t\mathrm{d}j + \int_0^1 \Psi_t\mathrm{d}j
\end{aligned}
$$

Using housing market clearing condition, rewrite the above expression:

$$
\begin{aligned}
&C_t^P + C_t^E + I_t - Y_t + Q_t\Big(\left(H - H_t^E\right) - \left(H - H_{t-1}^E\right)\Big) + Q_t^H\left(H_t^E - H_{t-1}^E\right) \\
&+ \int_0^1\int_0^1 D_{ik,t}\mathrm{d}i\mathrm{d}k + R_{t-1}^L\int_0^1\int_0^1 l_{jk,t-1}\mathrm{d}k\mathrm{d}j = \int_0^1\int_0^1 \Pi_{ik,t}\mathrm{d}k\mathrm{d}i \\
&+ R_{t-1}^D\int_0^1\int_0^1 D_{ik,t-1}\mathrm{d}k\mathrm{d}i + \int_0^1 x_{j,t}\mathrm{d}j + \int_0^1 \Phi_t\mathrm{d}j + \int_0^1 \Psi_t\mathrm{d}j
\end{aligned}
$$

After cancelling terms using the resource constraint, I now plug the expressions for $x_{j,t}$, Φ_t and $\Pi_{k,t}$ from the main text:

$$
\begin{aligned}
&\int_0^1\int_0^1 D_{ik,t}\mathrm{d}i\mathrm{d}k = R_{t-1}^D\int_0^1\int_0^1 D_{ik,t-1}\mathrm{d}k\mathrm{d}i - R_{t-1}^L\int_0^1\int_0^1 l_{jk,t-1}\mathrm{d}k\mathrm{d}j \\
&+ \int_0^1\left[\int_0^1\left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}}\mathrm{d}k\right]^{\frac{\xi}{\xi-1}}\mathrm{d}j + \gamma^L\int_0^1\int_0^1 \frac{\theta_{k,t}}{\theta_t}s_{k,t-1}\mathrm{d}k\mathrm{d}j + \Psi_t\mathrm{d}j \\
&+ \int_0^1\int_0^1\left(p_{k,t-1}R_{t-1}^L L_{k,t-1} + \left(1 - p_{k,t-1}\right)\frac{L_{k,t-1}}{\int_0^1 L_{k,t-1}\mathrm{d}k}\tau\theta_{t-1}a_{t-1} - L_{k,t}\right)\mathrm{d}k\mathrm{d}i \\
&+ \int_0^1\int_0^1\left(\int_0^1 D_{ik,t}\mathrm{d}i - R_{t-1}^D\int_0^1 D_{ik,t-1}\mathrm{d}i\right)\mathrm{d}k\mathrm{d}i
\end{aligned}
$$

Letting $\xi \to \infty$ and simplifying:

$$
\begin{aligned}
\int_0^1\int_0^1 D_{ik,t}\mathrm{d}i\mathrm{d}k = &R_{t-1}^D\int_0^1\int_0^1 D_{ik,t-1}\mathrm{d}k\mathrm{d}i - R_{t-1}^L\int_0^1\int_0^1 l_{jk,t-1}\mathrm{d}k\mathrm{d}j + \int_0^1\int_0^1\left(l_{jk,t} - \gamma^L s_{k,t-1}\right)\mathrm{d}k\mathrm{d}j \\
&+ \gamma^L\int_0^1\int_0^1 \frac{\theta_{k,t}}{\theta_t}s_{k,t-1}\mathrm{d}k\mathrm{d}j + \int_0^1 \Psi_t\mathrm{d}j - R_{t-1}^D\int_0^1\int_0^1 D_{ik,t-1}\mathrm{d}i\mathrm{d}k \\
&+ \int_0^1\int_0^1\left(p_{k,t-1}R_{t-1}^L L_{k,t-1} + \left(1 - p_{k,t-1}\right)\frac{L_{k,t-1}}{\int_0^1 L_{k,t-1}\mathrm{d}k}\tau\theta_{t-1}a_{t-1}\right)\mathrm{d}k\mathrm{d}i
\end{aligned}
$$

Cancelling terms and further simplifying:

$$
\begin{aligned}
\int_0^1\int_0^1 D_{ik,t}\mathrm{d}i\mathrm{d}k = &-R_{t-1}^L\int_0^1\int_0^1 l_{jk,t-1}\mathrm{d}k\mathrm{d}j + \int_0^1\int_0^1\left(l_{jk,t} - \gamma^L s_{k,t-1} + \gamma^L\frac{\theta_{k,t}}{\theta_t}s_{k,t-1}\right)\mathrm{d}k\mathrm{d}j \\
&+ \int_0^1 \Psi_t\mathrm{d}j + \int_0^1\left(1 - p_{k,t-1}\right)\frac{L_{k,t-1}}{\int_0^1 L_{k,t-1}\mathrm{d}k}\tau\theta_{t-1}a_{t-1}\mathrm{d}k + \int_0^1 p_{k,t-1}R_{t-1}^L L_{k,t-1}\mathrm{d}k
\end{aligned}
$$

Cancelling yet more terms and after simplifying more:

$$\int_0^1 \int_0^1 D_{ik,t} \mathrm{d}i \mathrm{d}k = -R_{t-1}^L \int_0^1 L_{k,t-1} \mathrm{d}k + \int_0^1 \int_0^1 l_{jk,t} \mathrm{d}k \mathrm{d}j + \int_0^1 \Psi_t \mathrm{d}j$$
$$+ \int_0^1 \Big(1 - p_{k,t-1}\Big) \tau \theta_{t-1} a_{t-1} \mathrm{d}k + R_{t-1}^L \int_0^1 p_{k,t-1} L_{k,t-1} \mathrm{d}k$$

After moving some terms around:

$$\int_0^1 \left(\int_0^1 D_{ik,t} \mathrm{d}i - L_{k,t} \right) \mathrm{d}k = \int_0^1 \Psi_t \mathrm{d}m + \int_0^1 \Big(1 - p_{k,t-1}\Big) \tau \theta_{t-1} a_{t-1} \mathrm{d}k - \int_0^1 \Big(1 - p_{k,t-1}\Big) R_{t-1}^L L_{k,t-1} \mathrm{d}k$$

Due to bank's balance sheet identity, the LHS becomes zero and I now have

$$\int_0^1 \Big(1 - p_{k,t-1}\Big) R_{t-1}^L L_{k,t-1} \mathrm{d}k - \int_0^1 \Big(1 - p_{k,t-1}\Big) \tau \theta_{t-1} a_{t-1} \mathrm{d}k = \int_0^1 \Psi_t \mathrm{d}j$$

Finally,

$$\int_0^1 \Psi_t \mathrm{d}j = \Psi_t = \int_0^1 \Big(1 - p_{k,t-1}\Big) \Big(R_{t-1}^L L_{k,t-1} - \tau \theta_{t-1} a_{t-1}\Big) \mathrm{d}k$$

where Fubini's theorem has been used to switch the order of integrals where necessary.