Credit Shocks, Lending Relationships and Economic Activity

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Abstract

This paper offers a theoretical framework to study the macroeconomics effects of a sudden spike in loans relative to deposits, or credit shocks, in an environment in which banks have endogenously-persistent lending relationships with their borrowers. The contribution of this work is to provide a setup that shows how credit shocks interact with bank-firm lending relationships, affect economic activity and drive macroeconomic fluctuations. In presence of lending relationships, a positive credit shock in this model leads to massive amplification of macroeconomic volatility which is absent when lending relationships are turned off. These effects are increasing in the degree of intensity and level of persistence of lending relationships. Higher volatility and persistence of credit shocks leads to increased amplification of macroeconomic volatility which is further magnified by presence of lending relationships.

Keywords: Credit Shocks, Loan-to-Deposit Ratio, Lending Relationships, Macroeconomic Fluctuations

JEL Classification: E32, E44

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1 Introduction

The contribution of this paper is to provide a framework to examine macroeconomic effects of credit shocks in a model in which bank-firm lending relationships matter. In this model, patient households supply deposits to banks which lend them to collateral-constrained entrepreneurs who own and run all the firms in the economy. These borrowers, over time, form endogenously-persistent lending relationships with their lenders which gives banks market power in loans market. A positive credit shock in this model, defined as an increase in loan-to-deposit ratio, increases economic activity and generates macroeocnomic fluctuations, over and beyond what can be observed in a model that abstracts from lending relationships. In this sense, this paper highlights the important role of bank market power in loans market and their part in shaping macroeocnomic dynamics when a credit shock hits the economy.

A positive loan-to-deposit shock can be interpreted as an increase in liquidity provision by the banking sector. This can happen, for example, due to an increase in the available capital in the economy and an expansion in investment and output (Pesaran and Xu, 2016). As reported by (Pesaran and Xu, 2016), the loan-to-deposit ratio has historically fluctuated around one. These fluctuations can be attributed to a series of factors. As emphasized by Pesaran and Xu (2016), from a liquidity perspective, fluctuations in loan-to-deposit ratios reflect funding mix of banks between retail and wholesale funding markets. The loan-to-deposit ratio tends to rise during good times when easy and cheap funding is abundantly available to finance credit growth and usually peters out when market conditions become stressed, when wholesale funding is substituted for retail savings and credit growth slows down.

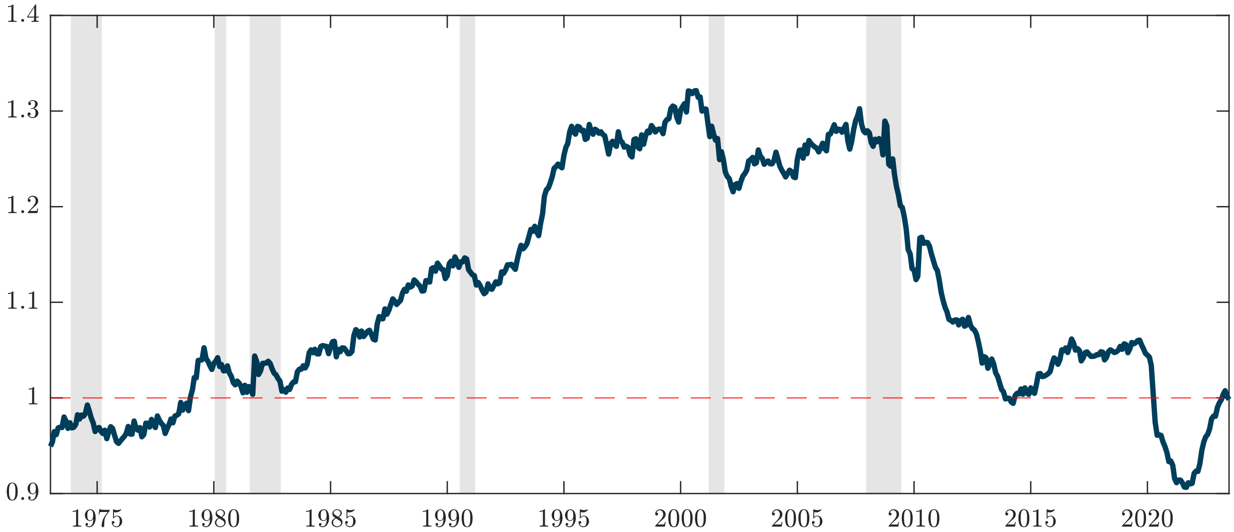
In order to investigate the macroeconomic effects of a credit shock in a model that takes into account presence of bank-firm lending relationships, I build a simple model featuring a households and a collateral-constrained entrepreneur. Households make deposits with banks which use it to make loans to firms run by entrepreneurs. These banks have credit relationships with banks. A credit shock in this model raises the amount of loans relative to deposits or loan-to-deposit ratio in the economy. In absence of any lending relationships between lenders and borrowers, these shocks have little effect on economic variables. However, they generate interesting economic movements which are interesting to examine and are quite different from the case when the economy features bank-firm credit relationships.

Absent credit relationships, a positive credit shock results in a rise in spread and fall in bank credit. This leads to a drop in investment, capital, output, labor and wages. Aggregate consumption, as a consequence, falls. A sudden spike in loans allows banks to seek higher margins to increase their profits which increases the spread between loan rate and deposit rate. This spurt in spread decreases the demand for loans and a slump in macroeconomic activity. This finding is quite different from what one might guess would happen in such a environment. This result, however, is turned upside down when one considers the existence of borrower-lender relationships. In this case, a credit shock leads to a significant drop in spread which reduces the cost of bank credit. This results in an investment boom that manifests in an increase in capital, outut, labor and wages. After an initial fall, spread returns to its prior equilibrium value before overshooting it and then it remains elevated for an extended period. In fact, it does not return to its previous steady state value after overshooting it until a decade (40 quarters). This reflects banks motive to seek higher profits from their customers after acquiring them. Bank know that their borrowers have deep habits in borrowing from them and they seek to use this relationship to gain higher profits. As a consequence of bank charging higher spread for a prolonged period, loans fall and remain below their steady state value for a long time. This leads to a fall in investment and capital which then feeds into a drop in labor, wages, and aggegate consumption. After some time, when spread and bank credit return to their previous steay state, investment, capital and other macroeconomic variables start recovering and begin to to return to their respective steady states. This highlights how presence of bank-firm credit relationships can act as an amplifier of credit shocks and a model that assumes away these lending relationships gives very different results. The magnitude of effects also is several order of magnitude higher when one considers lednding relationships. This underscors that presence of credit relationships can play an important role in amplifying macroeconomic fluctuations.

I further find that higher intensity and persistence of lending relationships magnify the effects of a credit shocks. The reason is that increased intensity of lending relationships allows banks to capture higher rent after a credit shock which translates into higher fall in spread after a credit shock which results into a greater increase in macroeconomic variables and then greater fall in economic activity when spread returns to steady state and overshoots is by a larger magnitude and then stays far longer above its prior equilibrium value. The same mechanism operates at greater persistence of lending relationships. This supports the conclusion that presence of credit relationships acts as an accelerator of macroeconomic fluctuations and echoes of "financial accelerator" effects of Bernanke, Gertler, and Gilchrist (1999).

In further show that credit shocks have the same amplifying effects at varying loan-to-deposit ratios. I consider three differnt LTD ratios and show that credit shocks have higher impact at higher LTD ratios, though their effect is qualitatively similar also at lower LTD ratios. Finally, I conduct two experiments. In the first experiment, I increase the volatility of credit shocks and show that their effects are increasing in the volatility of credit shocks. I then perform a second experiment in which I lower the persistence of credit shocks from its baseline and show that their effects decrease as persistence of credit shocks go down. This shows that effects of credit shocks are increasing in their volatility and persistence.

Figure 1: Loan to Deposit Ratio in the US



Note: Loans data is taken from the "US Commercial Bank Assets-Bank Credit " series from the Federal Reserve H.8 Table, comprising securities, loans and leases from all commercial banks in the US, among which, loans and leases include commercial and industrial loans, real estate and commercial loans. The Federal Reserve series "US Commercial Bank Liabilities-Deposits and Borrowing" (H.8 Table) is used as a measure for bank deposits, which captures large time deposits and other time deposits for all commercial banks. Shaded areas refer to NBER recession dates.

This is the first paper that takes into account existence of bank-firm lending relationships when examining macroeconomic implications of a positive credit shock. In doing so, it connects with two strands of literature. On one hand, it contributes to existing literature on economic effects of credit shocks, on the other hand it extends and builds on previous work in the area of bank-firm lending relationships and their macroeconomic implications. Prominent examples of recent work on effects of credit shocks are Pesaran and Xu (2016), Jensen, Ravn, and Santoro (2018) and Jensen, Petrella, Ravn, and Santoro (2020), among other. However, with the exception of Pesaran and Xu (2016), no other paper examines the macroeconomic effects of a shock to loan-to-deposit ratio which is the focus of this paper. Pesaran and Xu (2016) study the effects of a credit shock in an environment of firm default and with no presence of lending relationships between lenders and borrowers. My paper, on the other hand, abstracts from firm default and focuses on effects of a credit shock in the presence of bank-form lending relationships.

Many papers such as Ongena and Smith (2000) and Kosekova, Maddaloni, Papoutsi, and Schivardi (2023) have documented that firms in many countries form bank relationships over a period of time. However, no work exists that shows how presence of these lending relationships affect the implicatins of a credit shock. This paper fills this gap in the literature.

The other body of work that my paper connects to is existence of bank-firm lending relationships and their implications for macroeconomc dynamics. A number of papers have taken into consideration the presence of these lending relationships and have attempted to examine how the presence and nature of these lending relationships can affect various macroeconomic dynamics. An indicative list of papers in this literature include, among others, Aliaga-Díaz and Olivero (2010), Ravn (2016), Airaudo and Olivero (2019), Shapiro and Olivero (2020) and Sharma (2023c,a,d). None of these papers study the effects of a credit shock, modelled as a sudden rise in bank loans relative to deposits, in an environment of lending relationships. Sharma (2023b) studies the effects of credit shocks in a model in which banks compete on both interest rates and collateral requirements and the economy features a lending standard. This paper, in contrast, abstracts from collateral competition and instead focuses on bank competition on interest rates. The objective behind doing this is to keep the analysis focused on macroeconomic consequences of credit shocks in the simplest possible framework featuring bank-firm lending relationships and to shine a light on underlying mechanism without additional forces at play. Another paper this work is connected to is Sharma (2023e) who examines changes in lending conditions by looking at state dependence in loan-to-value (LTV) shocks. That paper, however, does not consider bank-firm lending relationships or credit shocks.

The rest of this paper is structured as follows. Section 2 presents the model and Section 3 discusses model solution and parameterization. Section 4 presents results and Section 5 concludes.

2 Model

The paper features a Two Agent RBC model and bears resemblance to the setup in Iacoviello (2005), Liu, Wang, and Zha (2013) and Justiniano, Primiceri, and Tambalotti (2015). The departure from the models in these papers is inclusion of a formal financial sector and presence of lending relationships between firms and banks.

There are two types of agents. The first type of agents are (patient) households who consume, supply labor, make deposits with a bank and receive profits from the firms they own. The second type of agents are (impatient) entrepreneurs who consume non-durable consumption good and run firms in the economy. They are subject to a collateral constraint which limits their borrowing to a fraction of expected value of theit assets which include productive capital and (durable) land. The entrepreneurs borrow from banks and develop endogenously-persistent credit relationships with them. Lending relationships in this paper are modelled by using the deep habits framework developed first by Ravn, Schmitt-Grohé, and Uribe (2006) and used later in studying banking sector by Aliaga-Díaz and Olivero (2010), Airaudo and Olivero (2019) and Shapiro and Olivero (2020), among others. These banks raise deposits from households which is their only source of funding and lend them to entrepreneurs who combine them with productive capital to produce output. In what follows, I describe each agent's optimization problem.

2.1 Households

Households have the utility function of the following form:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^P\right)^t \left\{ \log\left(C_{i,t}^P - \gamma^P C_{i,t-1}^P\right) - \frac{N_{i,t}^{\eta}}{\eta} + \varsigma \log H_{i,t}^P \right\} \tag{1}$$

where $C_{i,t}^P$, $N_{i,t}$ and $H_{i,t}^P$ denote consumption, labor and housing respectively of the households, $\beta^P \in (0,1)$ is a discount factor, γ^P measures the degree of habit formation in consumption, η is Frisch elasticity of labor supply and ς is a weight on housing. The superscript P denotes (patient) households. The household faces the following budget constraint

$$C_{i,t}^P + Q_t^H \left(H_{i,t}^P - H_{i,t-1}^P\right) + \int_0^1 D_{ik,t} \mathrm{d}k \leq W_t N_{i,t} + \int_0^1 \Pi_{ik,t} \mathrm{d}k + R_{t-1}^D \int_0^1 D_{ik,t-1} \mathrm{d}k \tag{2}$$

where Q_t^H is the price of one unit of housing in terms of consumption goods, W_t is the real wage and R_{t-1}^D is the gross risk-free interest rate on the stock of deposits $D_{ik,t-1}$ of household i in bank k at the end of period $t-1$. I assume housing does not depreciate. Profits obtained by household i from bank k are denoted by $\Pi_{ik,t}$. After imposing symmetric equilibrium, FOCs of the households can be written as

$$\frac{1}{C_t^P - \gamma^P C_{t-1}^P} - \beta^P \mathbb{E}_t \frac{\gamma^P}{C_{t+1}^P - \gamma^P C_t^P} = \lambda_t^P \tag{3}$$

$$\beta^P \mathbb{E}_t \lambda_{t+1}^P = \frac{\lambda_t^P}{R_t^D} \tag{4}$$

$$\frac{\varsigma}{H_t^P} + \beta^P \mathbb{E}_t \left(\lambda_{t+1}^P Q_{t+1}^H\right) = \lambda_t^P Q_t^H \tag{5}$$

$$N_t^{\eta-1} = \lambda_t^P W_t \tag{6}$$

where λ_t^P is the Lagrange multiplier associated with household's budget constraint (2). One can combine household's first-order conditions with respect to consumption (3) and bank deposits (4) to obtain their Euler equation. Equation (5) describes household's Euler equation for housing and links today's housing price to the utility it provides plus the expected capital gain. Equation (6) describes household's consumption-lesiure tradeoff. First order conditions of the problem are derived in the Appendix A.1.

2.2 Entrepreneurs

Following Iacoviello (2005) and Liu, Wang, and Zha (2013), entrepreneur j maximizes the utility obtained from consuming the non-durable consumption goods

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^E\right)^t \log\left(C_{j,t}^E - \gamma^E C_{j,t-1}^E\right) \tag{7}$$

where β^E and γ^E are as defined before. I assume that entrepreneurs are more impatient than the (patient) households, that is, $\beta^E < \beta^P$. Entrepreneurs face a collateral constraint as in Kiyotaki and Moore (1997) that limits the borrowing of each entrepreneur to a fraction of his assets

$$l_{jk,t} \leq \frac{1}{R_{k,t}^L} \theta a_{j,t} \tag{8}$$

Here, $l_{jk,t}$ denotes entrepreneur j's loan from bank k, expected value of entrepreneur's assets is $a_{j,t}$ and $R_{k,t}^L$ is the bank-specific lending rate. All entrepreneurial borrowing is subject to a loan-to-value (LTV) requirement θ. Expected valued of entrepreneur's assets , $a_{j,t}$ is given by

$$a_{j,t} = \mathbb{E}_t \left(Q_{t+1}^H H_{j,t}^E + Q_{t+1}^K K_{j,t}\right) \tag{9}$$

In the above equation, Q_t^K denotes the value of installed capital in units of consumption goods, $K_{j,t}$ stock of capital and $H_{j,t}^E$ stock of housing.

Entrepreneurs have deep habits in banking relationships and and we let $x_{j,t}$ denote entrepreneur j's effective/habit-adjusted borrowing. Given the continuum of banks in the economy who compete under monopolistic competition, this can be written as

$$x_{j,t} = \left[\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1} \right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right]^{\frac{\xi}{\xi-1}} \tag{10}$$

where stock of habits $s_{k,t-1}$ evolves according to

$$s_{k,t-1} = \rho_s s_{k,t-2} + (1-\rho_s)\, l_{k,t-1} \tag{11}$$

Here, $\gamma^L \in (0,1)$ denotes the degree of habit formation in demand for loans and $\rho_s \in (0,1)$ measures the persistence of this habits. The parameter ξ denotes of the elasticity of substitution between loans from different banks and is thus a measure of the market power of each individual bank.

Given his total need for financing $x_{j,t}$, each entrepreneur chooses $l_{jk,t}$ to solve the following problem

$$\min_{l_{jk,t}} \int_0^1 R_{k,t} l_{jk,t} \mathrm{d}k \tag{12}$$

subject to collateral constraint (8) and his effective borrowing (10). Entrepreneur j's optimal demand for loans from bank k is

$$l_{jk,t} = \left(\frac{R_{k,t}}{R_t} \right)^{-\xi} x_t + \gamma^L s_{k,t-1} \tag{13}$$

where $R_t^L \equiv \left[\int_0^1 (R_{k,t}^L)^{1-\xi} \mathrm{d}k \right]^{\frac{1}{1-\xi}}$ is the aggregate lending rate. Production function of each entrepreneur is

$$Y_{j,t} = A_t \left(N_{j,t} \right)^{1-\alpha} \left\{ \left(H_{j,t-1}^E \right)^{\phi} \left(K_{j,t-1} \right)^{1-\phi} \right\}^{\alpha} \tag{14}$$

where $Y_{j,t}$ is output, $N_{i,t}$ is labor input and $\alpha, \phi \in (0,1)$ are factor shares. TFP A_t follows the process

$$\log A_t = (1-\rho_A) \log A + \rho_A \log A_{t-1} + \sigma_A \epsilon_{A,t} \tag{15}$$

with iid innovation $\epsilon_{A,t}$ following a normal process with standard deviation σ_A where $A > 0$ and $\rho_A \in (0,1)$. The evolution of capital obeys the following law of motion

$$K_{j,t} = (1-\delta) K_{j,t-1} + \left[1 - \frac{\Omega}{2}\left(\frac{I_{j,t}}{I_{j,t-1}} - 1\right)^2\right] I_{j,t} \tag{16}$$

where $I_{j,t}$ is firm j's investment level, $\delta \in (0,1)$ the rate of depreciation of capital stock and $\Omega > 0$ is a cost adjustment parameter. The entrepreneur faces the following budget constraint

$$C^E_{j,t} + \int_0^1 R^L_{k,t-1} l_{jk,t-1} \mathrm{d}k \leq Y_{j,t} - W_t N_{j,t} - I_{j,t} - Q^H_t \left(H^E_{j,t} - H^E_{j,t-1}\right) + x_{j,t} \tag{17}$$

After imposing symmetric equilibrium, the FOCs of the entrepreneurs are

$$\lambda^E_t = \frac{1}{C^E_t - \gamma^E C^E_{t-1}} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C^E_{t+1} - \gamma^E C^E_t} \tag{18}$$

$$\lambda^E_t = \beta^E \mathbb{E}_t \lambda^E_{t+1} R^L_t + \mu^E_t R^L_t \tag{19}$$

$$W_t = (1-\alpha) \frac{Y_t}{N_t} \tag{20}$$

$$\lambda^E_t Q^H_t = \beta^E \mathbb{E}_t \left[\lambda^E_{t+1} \left(Q^H_{t+1} + \alpha\phi \frac{Y_{t+1}}{H^E_t}\right)\right] + \mu^E_t \theta_t \mathbb{E}_t Q^H_{t+1} \tag{21}$$

$$\kappa^E_t = \alpha (1-\phi) \beta^E \mathbb{E}_t \left(\frac{\lambda^E_{t+1} Y_{t+1}}{K_t}\right) + \beta^E (1-\delta) \mathbb{E}_t \kappa^E_{t+1} + \mu^E_t \theta_t \mathbb{E}_t Q^K_{t+1} \tag{22}$$

$$\lambda^E_t = \kappa^E_t \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \Omega \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1\right)\right] + \beta^E \Omega \mathbb{E}_t \left[\kappa^E_{t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 \left(\frac{I_{t+1}}{I_t} - 1\right)\right] \tag{23}$$

where μ^E_t, κ^E_t and λ^E_t are Lagrange multipliers associated with entrepreneur's collateral constraint (8), law of motion of capital (16) and entrepreneur's budget constraint (17). Entrepreneur's first order conditions with respect to consumption (18) and loans (19) may be combined to derive Euler equation for consumption for a collateral-constrained agent. Equation (20) describes entrepreneur's optimal demand for labor. Entrepreneur's Euler equation for land is described by (21) which relates its price today to its expected resale value tomorrow plus the payoff obatained by holding it for a period as given by its marginal producivity and its ability to serve as a collateral. Likewise, (22) is entrepreneur's Euler equation for capital and it links price of capital today to its price tomorrow and the expected payoff from keeping it for a period as given by its marginal productivity and its ability to serve as a collateral. Finally, entrepreneur's Euler equation for the investment is given by (23). All the derivations of first order conditions have been relegated to Appendix A.2

2.3 Banking Sector

Banks in this model accept deposits from households and make loans to entrepreneurs. Their balance sheet follows the structure in Freixas and Rochet (2023). The balance sheet of bank k is

$$L_{k,t} = \psi_t \int_0^1 D_{ik,t} \mathrm{d}i \tag{24}$$

where $L_{k,t}$ denotes total loans made by bank k to all entrepreneurs, that is, $L_{k,t} \equiv \int_0^1 l_{jk,t} \mathrm{d}j$ and ψ_t is an exogenous shock which obeys the following law of motion

$$\log \psi_t = (1 - \rho_\psi) \log \psi + \rho_\psi \log \psi_{t-1} + \sigma_\psi \epsilon_{\psi,t} \tag{25}$$

Equation (24) offers a tractable way to introduce shocks that originate on the supply side of bank's balance sheet and allow for an examination of how credit shocks transmit to the real economy. A brief discussion of the nature and meaning of credit shock is warranted here. Credit shocks can stand in for a number of factors in this setup. As Pesaran and Xu (2016) explain, this formulation can be interpreted as the requirement that banks are required to deposit some reserve B_t with the central bank. Central bank uses this requirement to influence the amount of credit in the economy. This will mean that $L_t + B_t = D_t$ where $B_t = (1 - \psi_t) D_t$. The purpose of this policy req can be as follows. When the economy is overheating and investment is high, the central bank can raise the reserve-requirement ratio $(1 - \psi_t)$ to curb credit expansion and reduce inflationary pressure in the economy, in which case the reserve requirement acts as a countercyclical policy tool. Alternatively, when credit risk is high (for instance, because of default risk which is not modelled in this paper), the central bank can raise the reserve-requirement ratio so that banks put aside sufficient reserve to cushion the effects of higher bank losses due to firm defaults. Additionally, ψ_t can be interpreted as a macroprudential policy tool where the financial regulatory tool targets the volume of loans extended to the real economy, to dampen the procyclicity in the credit cycle. In both these cases, ψ_t will be less than one.

However, it is also possible to consider the cases when ψ_t is greater than one. This is possible when banks are allowed to issue securities (IOUs) that are not backed by deposits. These securities could potentially be guaranteed by the central bank in event of a bank run (not modeled in our framework). The central bank can also be a source of additional liquidity to the banking sector, as seen in the recent financial crisis. To model this possibility explicitly, one would need to introduce the price level and inflation into our framework, since the central bank's credit provision could lead to inflationary pressure in the economy. Given the relatively simple and canonical characterization of the banking sector in our model, we abstract from pinpointing the exact source of the credit shock; instead, we investigate all three different scenarios where the mean of ψ_t is less than, equal to, and greater than unity in our calibration and simulation exercises. These three scenarios are motivated by time series evidence in the US, where the historical loan-to-deposit ratio has fluctuated around one.

Banks take the interest rate on deposits R_t^D as given. Each individual bank k chooses its lending rate $R_{k,t}^L$ and its total amount of lending $L_{k,t}$. Bank's profit function is

$$\Pi_{k,t} = R_{k,t-1}^L L_{k,t-1} + \int_0^1 D_{ik,t}\mathrm{d}i - L_{k,t} - R_{t-1}^D \int_0^1 D_{ik,t-1}\mathrm{d}i \tag{26}$$

Each bank takes the demand for its loans as given

$$L_{k,t} = \int_0^1 l_{jk,t}\mathrm{d}j = \int_0^1 \left[\left(\frac{R_{k,t}}{R_t} \right)^{-\xi} x_t + \gamma^L s_{k,t-1} \right] \mathrm{d}j \tag{27}$$

Each bank chooses $L_{k,t}$ and $R_{k,t}^L$ to maximize its profits subject to (24) and (27). Considering a symmetric equilibrium in which all banks optimally choose the same lending rate, the FOCs for banks' optimization problem are:

$$\varrho_t^E = \left(\frac{1}{\psi_t} - 1 \right) + \mathbb{E}_t q_{t,t+1} \left[\left(R_t^L - \frac{R_t^D}{\psi_t} \right) + \gamma^L (1 - \rho_s) \mathbb{E}_t \varrho_{t+1}^E \right] \tag{28}$$

and

$$\frac{1}{\psi_t} \xi \varrho_t^E x_t \frac{1}{R_t^L} = \mathbb{E}_t q_{t,t+1} L_{k,t} \tag{29}$$

where ϱ_t^E is the Lagrange multiplier on demand for bank's loans (27) and can be interpreted as shadow value to the bank of lending an extra dollar. Banks are owned by households and consequently their stochastic discount factor is given by $q_{t,t+1} = \beta^P \mathbb{E}_t \frac{\lambda_{t+1}^P}{\lambda_t^P}$. The optimality condition (28) states that shadow value of lending an extra dollar is given by repayment minus cost of borrowing that extra dollar from the households. The term $\gamma^L (1 - \rho_s) s_{k,t-1}$ on the right-hand side reflects the fact that if a given bank lends an extra dollar in this period, the borrower of that dollar will develop will develop a habit for loans from that bank and as a result, will borrow more from it also in the susbesequent period. The size of this effect depends on degree γ^L and duration ρ_s of deep habits. In absence of deep habits (when γ^L is zero), the latter term disappears. Equation (29) equates the profit gain from a marginal increase in bank's lending rate to the marginal cost. Bank's marginal cost is on the left-hand side and indicates a loss in its market share as it increases its lending rate. The marginal benefit of a higher lending rate appears on the right-hand side and shows the discounted gain made by repayment of loans made at higher lending rates. All the derivations are contained in Appendix A.3.

2.4 Aggregation and Market Clearing

Aggregate resource constraint of the economy is

$$C_t^P + C_t^E + I_t = Y_t \tag{30}$$

The clearing condition for the housing market is

$$H_t^P + H_t^E = H \tag{31}$$

where H is the total fixed supply of housing.

3 Model Solution and Prameterization

A period in the model refers to a quarter. Appendices B, C and D contain the list of equilibrium equations, the list of steady-state conditions and the system of log-linear equations, respectively. The calibration of parameters is rather standard and is summarized in Table 1. I allow for a relatively significant difference between discount factors of households and entrepreneurs so that steady-state value of μ_t^E is different from zero. The degree of habit formation in consumption is chosen to be 0.6 which is in line with empirical estimates (Smets and Wouters, 2007). The Frisch elasticity of labor supply η is chosen to be 1.01 and the value of weight on housing ς is set to 0.1 (Iacoviello, 2005).

The labor income share is 0.3 which implies a steady-state capital-output ratio of 1.15, in line with US data (Liu, Wang, and Zha, 2013). The input share of land in production is close to the

Table 1: Parameter values

	Value	Description	Source/Target
β^P	0.995	Discount factor, households	Iacoviello (2005)
β^E	0.95	Discount factor, entrepreneurs	Iacoviello (2005)
γ^i, $i = \{P, E\}$	0.6	Habits in consumption, households, entrepreneurs	Smets and Wouters (2007)
η	1.01	Frisch elasticity of labor	Iacoviello (2005)
ς	0.1	Weight on housing	Iacoviello (2005)
α	0.3	Non-labor share of production	See Text
ϕ	0.1	Land share of non-labor input	See Text
Ω	1.85	Investment adjustment cost parameter	See Text
δ	0.0285	Capital depreciation rate	See Text
γ^L	0.72	Deep habit formation	Aliaga-Díaz and Olivero (2010)
ρ_s	0.93	Persistence of stock of deep habits	See Text
ξ	230	Elasticity of substitution between banks	Ravn (2016)
ρ_A	0.95	Persistence of technology shock	Smets and Wouters (2007)
ρ_ψ	0.848	Persistence of credit shock	Pesaran and Xu (2016)
σ_A	0.0014	Standard deviation of technology shock	See Text
σ_ψ	0.011	Standard deviation of credit shock	Pesaran and Xu (2016)
ψ	1	Mean of loan-to-deposit ratio	See Text

value estimated in Liu, Wang, and Zha (2013) and Iacoviello (2005). The investment adjustment cost parameter is given a value of 1.85 (Ravn, 2016). The literaure contains estimates which range from 0 (Liu, Wang, and Zha, 2013) to above 26 (Christiano, Motto, and Rostagno, 2010). The capital depreciation rate implies a steady-state ratio of non-residential investment to output slightly above 0.13 as in Beaudry and Lahiri (2014).

For parameters in the banking sector, I rely on Aliaga-Díaz and Olivero (2010). I set the deep habit parameter in lending γ^L to 0.72, only as the baseline and later vary it to capture in a transparent fashion how it affects credit shocks. Similarly, I set the autocorrelation parameter in stock of habits in lending ρ_s to 0.93 which is close to the value of 0.85 used by both Ravn, Schmitt-Grohé, and Uribe (2006) and Aliaga-Díaz and Olivero (2010). This gives a bank-firm relationship of 11 years (Petersen and Rajan, 1995). I take this as baseline and later vary it to study the effects of lending relationship persistence on transmission of credit shocks. Specifically, I consider the effects of credit shocks when stock of habits is such that after 10 years, stock of habits left is 0% and 10%. Setting $\gamma^L = 0$ shuts off deep habits in banking and setting $\rho = 0.86$ implies that after 44 quarters, the stock of habits is zero. I run simulations in which I gradually lower γ^L from its baseline value and examine impulse response functions. I also conduct experiments in which I consider two other autocorrelation parameters $\rho_s = 0.86$ and

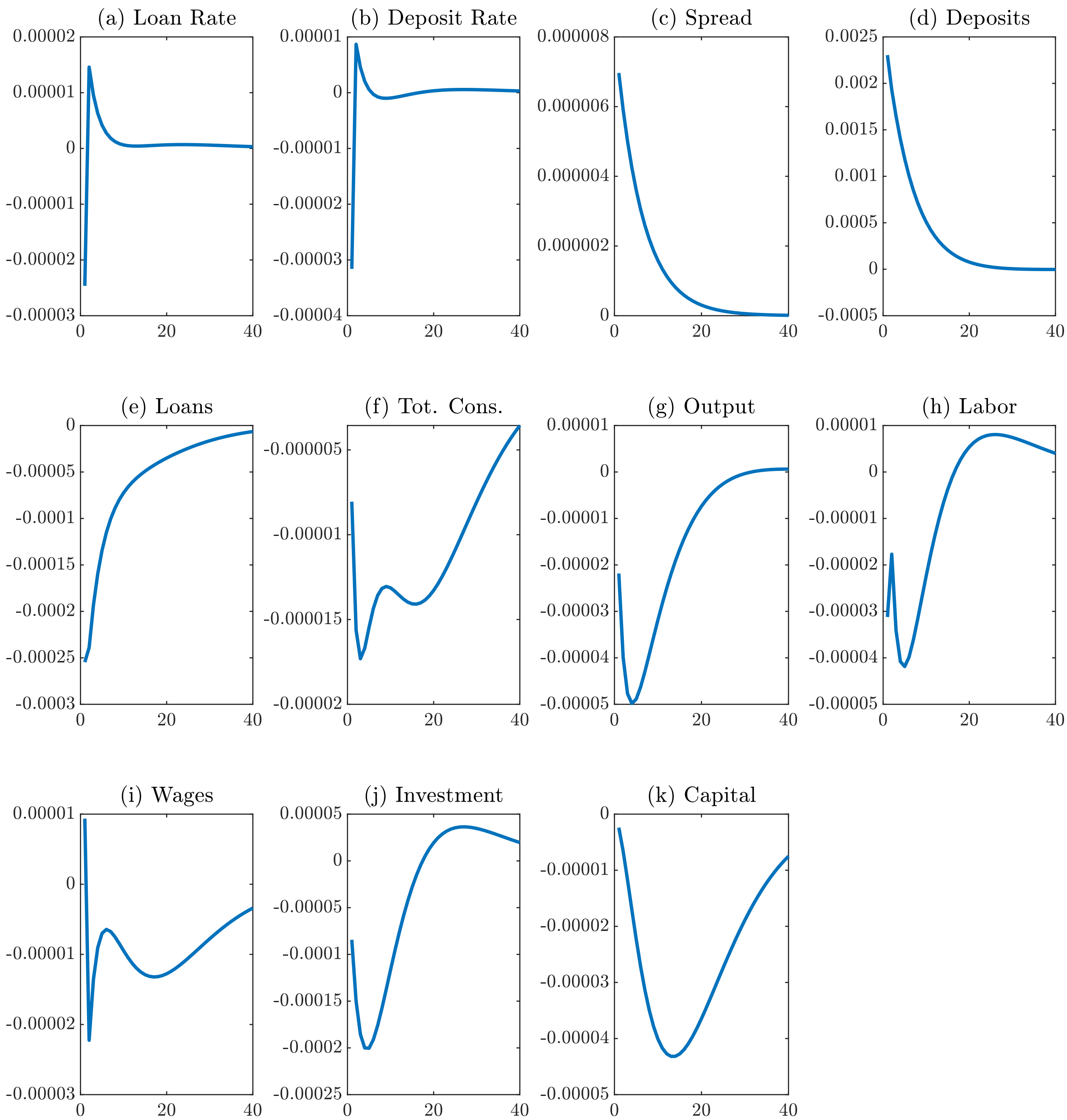
$\rho_s = 0.949$ denoting 0% and 10% stock of habit after 10 years, respectively. This allows me to investigate the impact of persistence of lending relationships on credit shocks. For elasticity of substitution between different loan varieties ξ, I pick the value as 230 which is close to the value of 190 used in Aliaga-Díaz and Olivero (2010) while Melina and Villa (2018) use a value of 427.

Following Smets and Wouters (2007), I set the persistence of TFP shock to 0.95 and its standard deviation to 0.0014 which is standard in the literature. For credit shocks, I follow Pesaran and Xu (2016) and set the volatility of credit shock σ_ψ to 0.011 and autocorrelation parameter of volatility shock ρ_ψ to 0.848. I call these values baseline. I later conduct experiments in which I increase volatility by 50% and reduce autocorrelation parameter by 20%. These experiments allow me to capture in a transparent fashion the effects of credit shocks.

4 Discussion

This section discusses the effects of a positive credit shock on macroeconomic activity. I begin by describing what happens in an economy in the wake of a credit shock absent lending relationships. Figure 2 shows the impulse responses of select variables. In the aftermath of a credit shock, the spread rises which makes borrowing more expensive and as a consequence, bank loans fall. This is followed by a fall in investment which reduces capital and output. Slump in investment and capital stock leads to a drop in labor and wages which brings down aggregate consumption. These effects are small but interestingly, as I will explain shortly, quite different from the case when the economy features lending relationships between lenders and borrowers. To see this clearly, I plot the impulse responses after a credit shocks for both cases – when lending relationships are present and when they are absent. As Figure 3 shows, after a positive credit shock, spread falls which makes bank credit less costly and loans increase as a result. This investment boom increases investment and capital stock. Aggregate ouptput and labor rises in tandem with wages which increases aggregate consumption. This behaviour is starkly different from the case in which the economy features no lending relationships. The fall in spread and increase in loans is much higher in presence of lending relationships. In the wake of a credit shock, labor rises which pushes wages down at impact before it rises in response to a fall in labor. Interestingly, consumption, output, investment and capital become more volatile when a credit shock hits the economy. This illustrates the importance of considering borrower-lender lending relationships, since omitting it might lead to underestimating the effects of a credit shock. The effects in the case of no lending relationships are so small that when plotted against the case of lending relationships in the same graph, they appear as almost horizontal line with little movement. This suggest an interesting result. It shows that a credit shock may not cause much economic movement in a simple model which does not have any borrower-lender relationships, but it can lead to enormously higher macroeconomic fluctuations when lending relationships are considered.

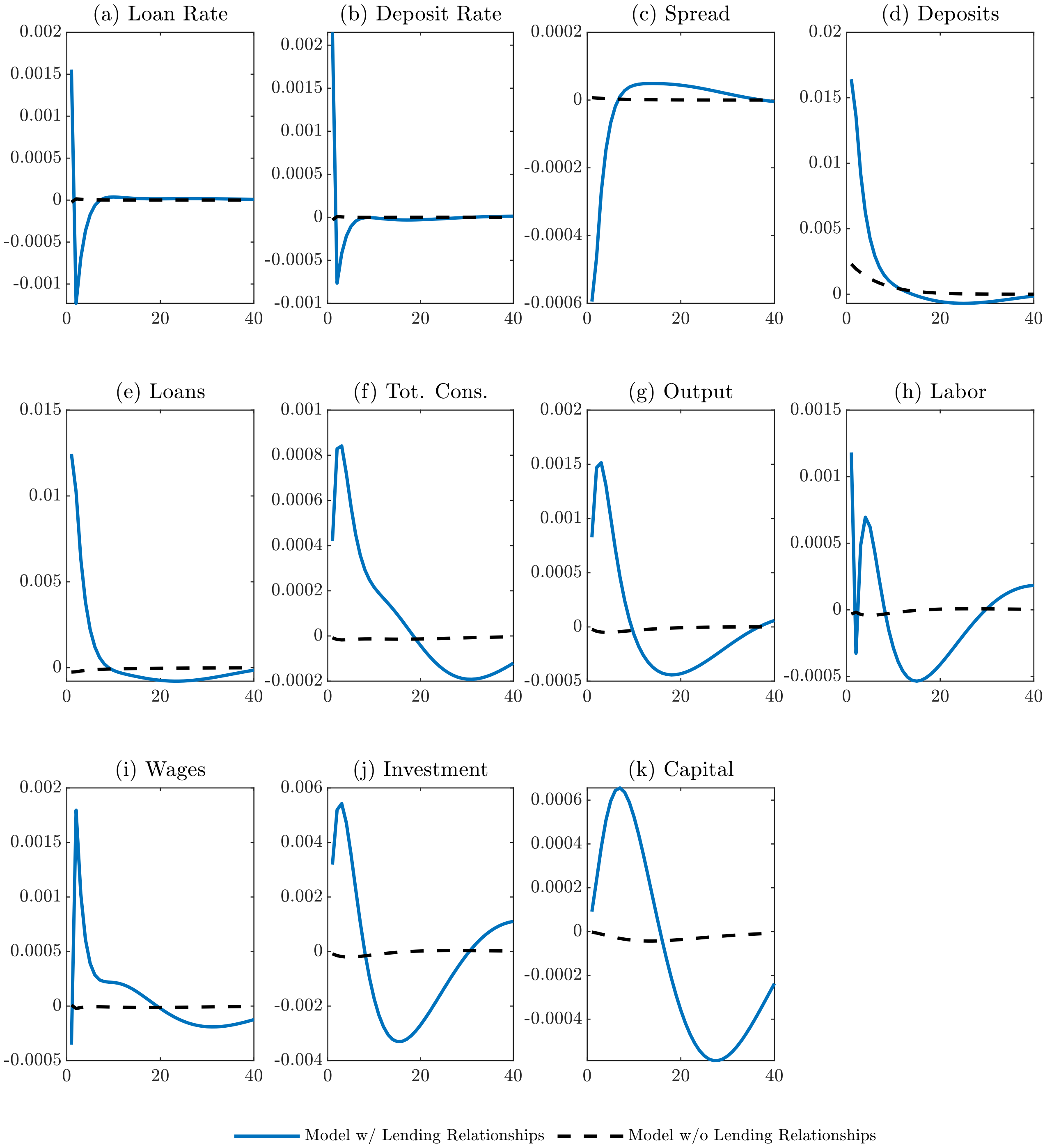
Figure 2: Impact of a Credit Shock in Absence of Lending Relationships



Note: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

Figure 4 show the effects of a credit shock at various degrees (levels of intensity) of lending relationships. I consider 0.72 as benchmark and then gradually reduce it to 0.62, 0.52 and 0.

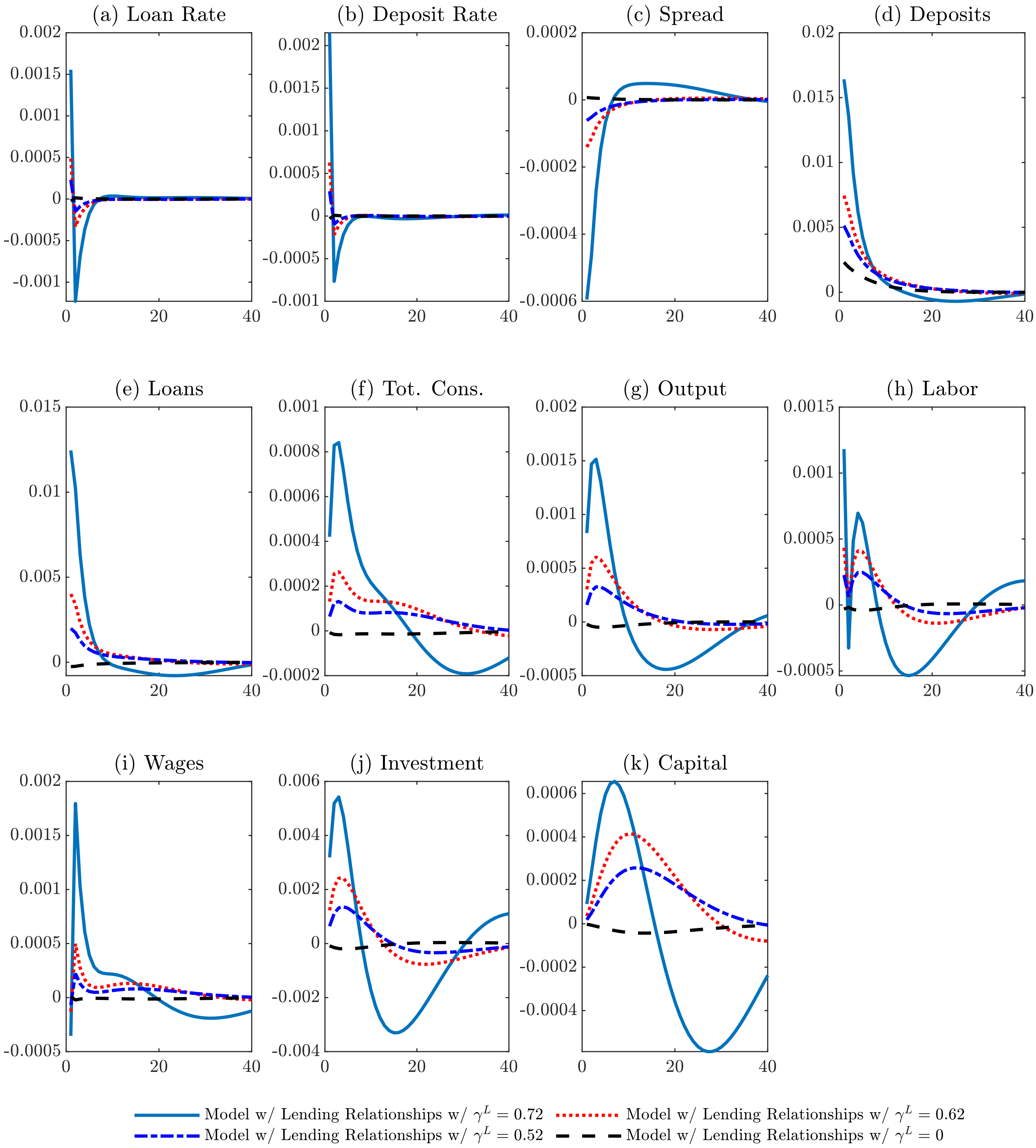
FIGURE 3: IMPACT OF A CREDIT SHOCK



NOTE: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

This exercise allows for a transparent examination of effects of a credit shock at various degrees of lending relationships. As one would expect, at degrees of lending relationships decrease, the volatility in macroeconomic variables decreases. When the degree of lending relationship equals 0 which corresponds to the case of no lending relationships, the macroeconomy shows very little volatility. This indicates that lending relationships act as an amplifier when credit shocks hit the economy and greater the intensity of lending relationships, the greater is the amplification of macroeocnomic aggregates in response to a credit shock.

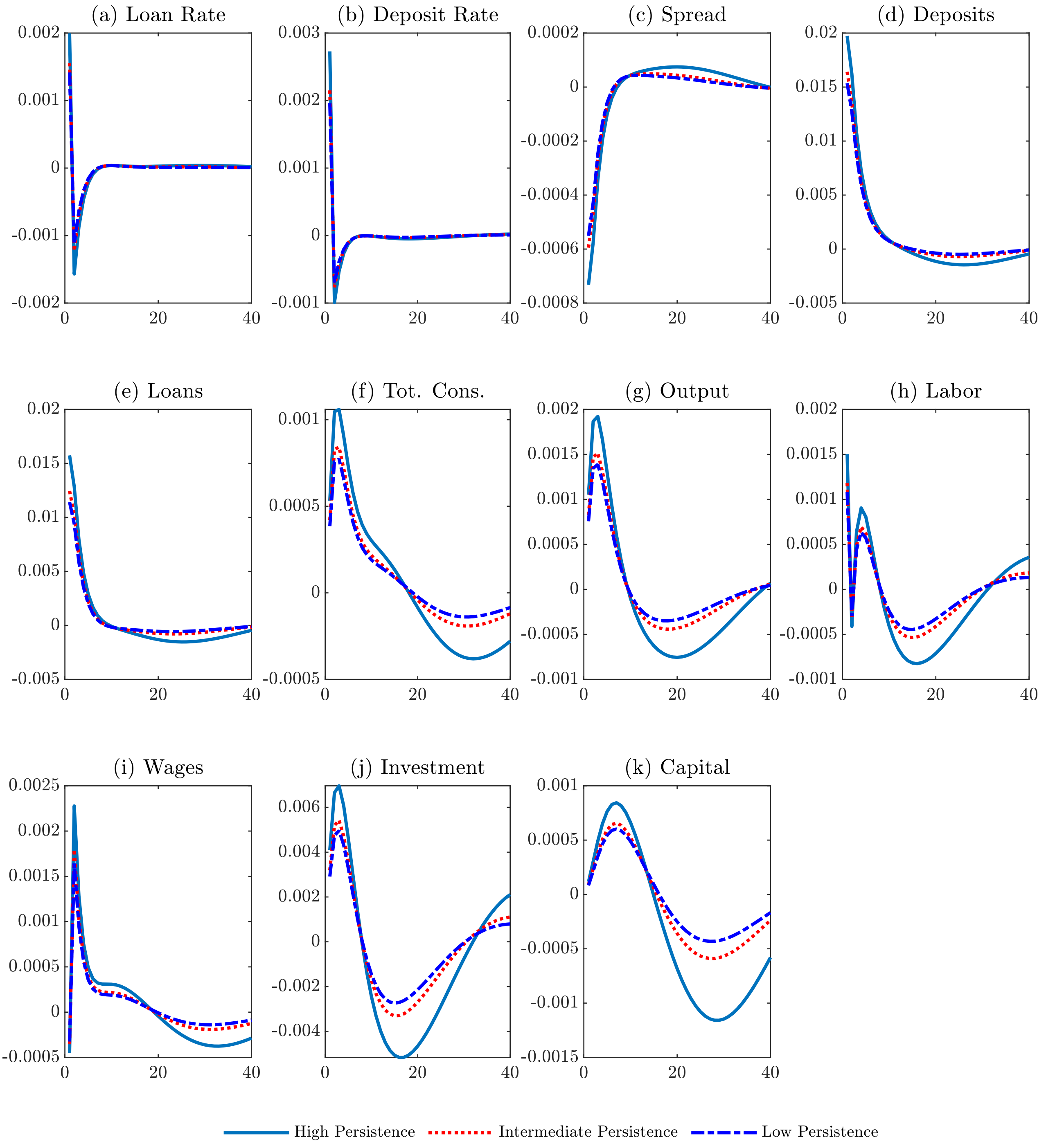
Figure 4: Impact of a Credit Shock: Different Degrees of Habits in Loans



Note: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

Figure 5 shows the effects of a credit shock at various levels of persistence of lending relationships. I consider three different cases – level of persistence when 10% stock of habits remains after a decade, when 5% stock of habits remains after 10 years and finally when 2.5% stock of habits remains after a decade. At higher persistence of lending relationships, the effects of credit shocks are amplified while lower persistence of lending relationships mutes the effects of the credit shock. It suggests that as persistence of lending relationships increase, the credit shocks become more amplified.

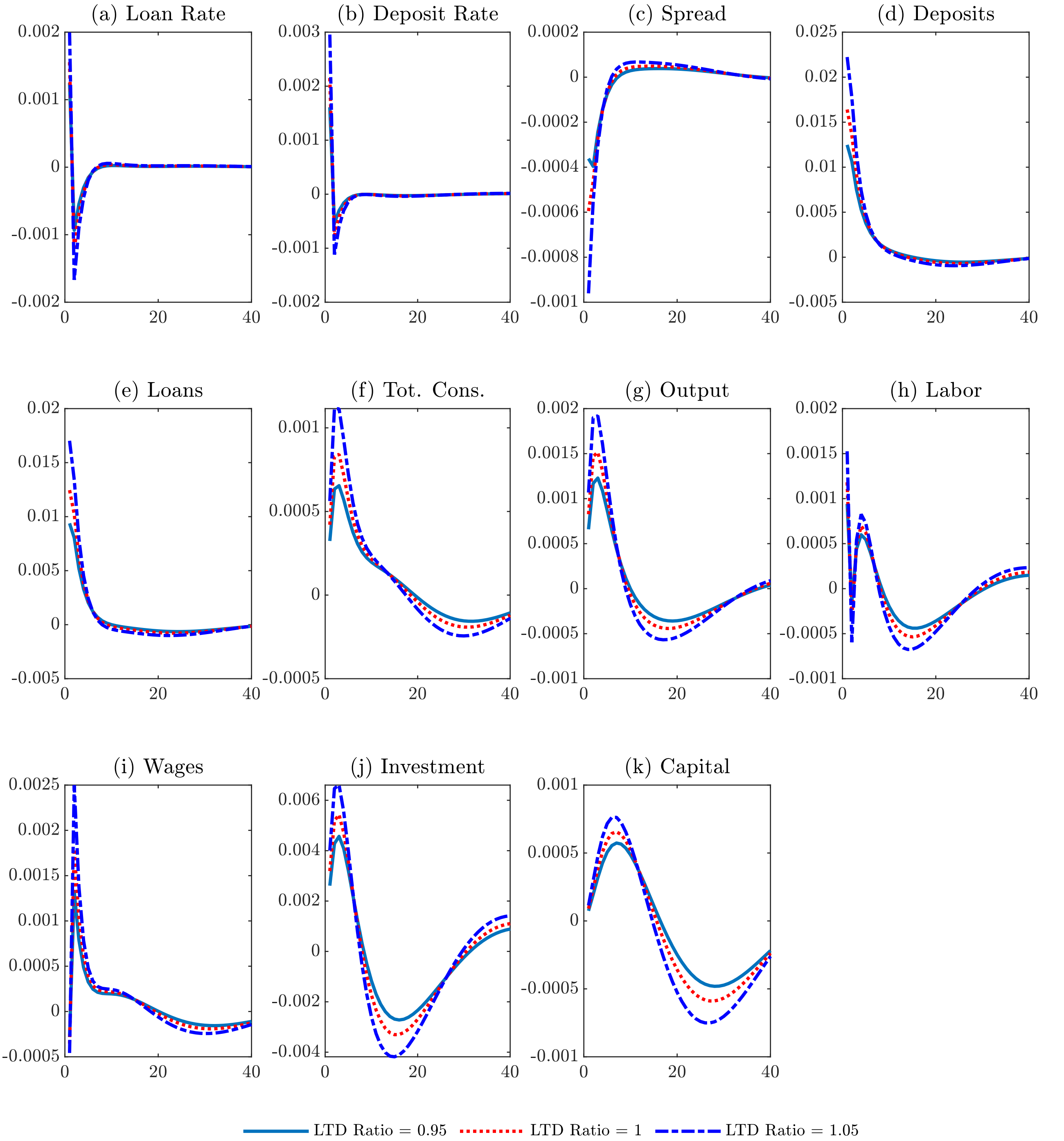
Figure 5: Impact of a Credit Shock: Different Levels of Persistence of Stock of Habits



Note: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

Figure 6 shows the effects of credit shocks at different loan-to-deposit (LTD) ratios. I conisder three LTD ratios of 0.95, 1 and 1.05 which correspond to the cases of when loans are less than deposits, when loans equal deposits and when loans exceed deposits. At every LTD ratio, credit shocks amplify macroeconomic volatility. These effects are greater at higher LTD ratios.

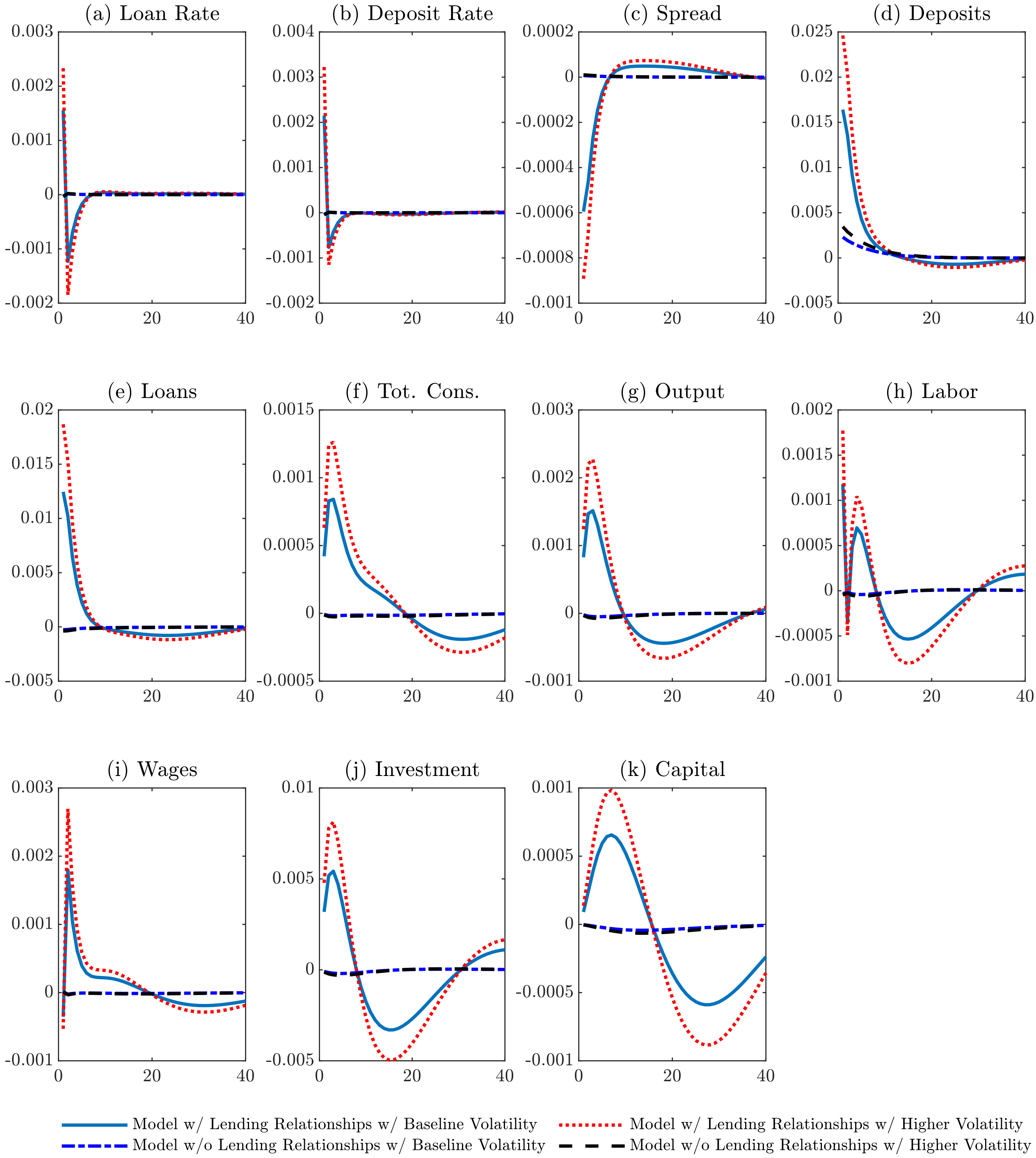
Figure 6: Impact of a Credit Shock at Different Loan-to-Deposit Ratios



Note: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

Figure 7 shows the differing effects of different volatilities of credit shocks on macroeconomic variables. I consider two scenarios. One in which I keep the baseline volatility used in main calibration and the other in which I raise the volatility of the credit shock by 50%. I do this exercise both for the case when lending relationships are present and when they are absent. At higher volatilities, the effects of credit shocks are amplified. Their effect is almost zero when there are no lending relationships. This ties in well with the earlier result in this paper that in presence of lending relationships, effects of credit shocks are greatly amplified and they are largely absent when the economy does not feature lending relationships.

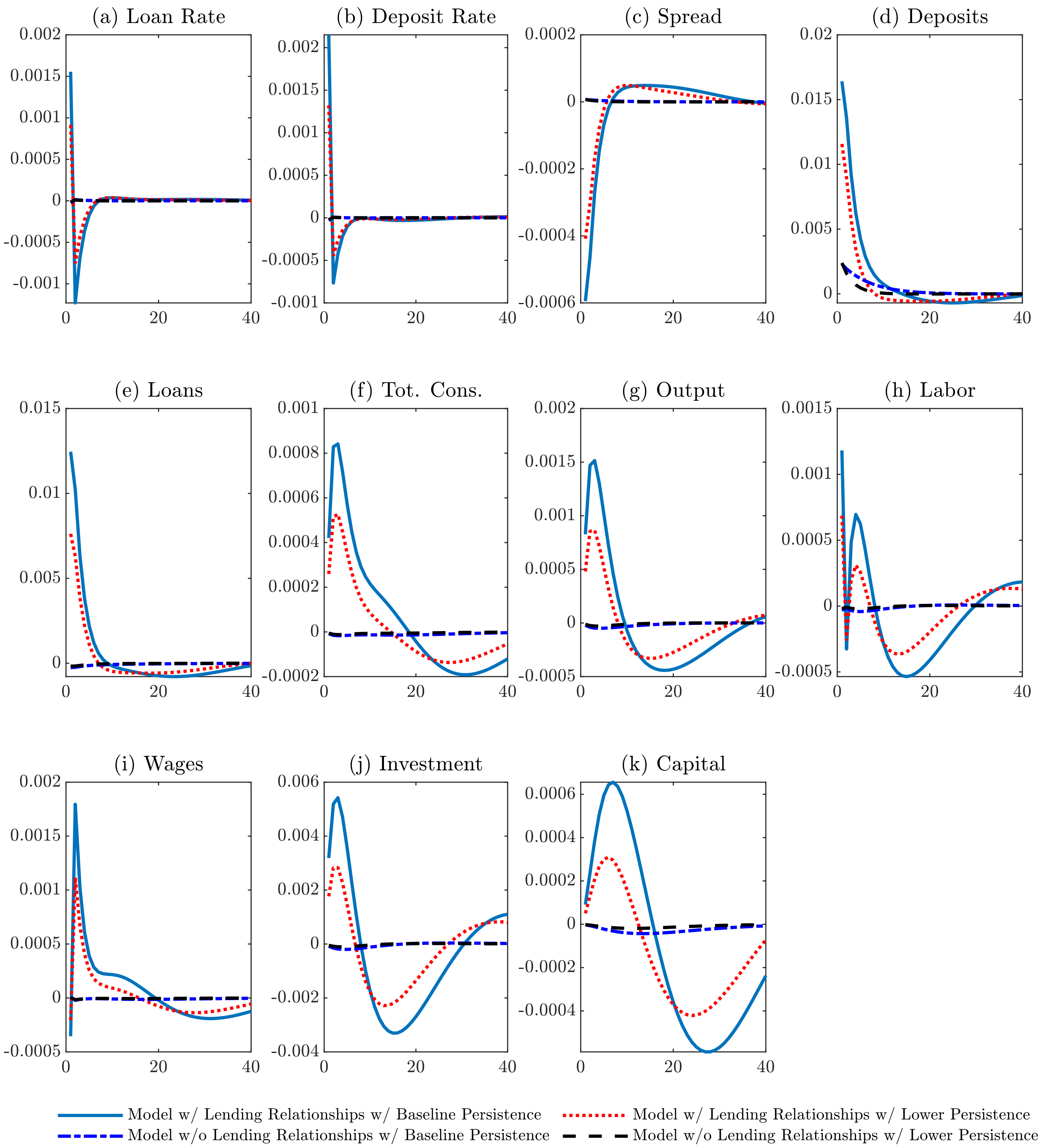
Figure 7: Impact of a Credit Shock at Different Volatilities



Note: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

Figure 8 demonstrates the effects of different levels of persistence of credit shock on macroeconomic vatriables. I consider two scenarios. One in which the persistence of credit shock is kept at the baseline and the other in which I lower the level of persistence by 20%. As persistence of the credit shock goes down, the effects of credit shock become muted. This effect is more pronounced in presence of lending relationships versus the case in which they are absent. This, again, confirms the earlier finding in this paper that lending relationships seems to have magnifying effect when a credit shock hits the economy.

FIGURE 8: IMPACT OF A CREDIT SHOCK AT DIFFERENT LEVELS OF PERISTENCE



NOTE: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

5 Conclusion

This paper studies the effects of a credit shock in a model in which borrowers have endogenously-persistent lending relationships with banks. I show that lending relationships have an amplifying effect on credit shocks, defined as sudden spike in loans relative to deposits. These effects are absent when the economy does not feature any lending relationships. The effects of credit shocks increase as intensity and persistence of lending relationships increase. Further, at higher volatility and persistence of credit shocks, their effects are higher. The key finding from this paper is that when studying effects of a credit shock, borrower-lender relationships matter and a model that abstracts from lending relationships in banking sector may end up grossly underestimating the true effects of a credit shock.

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Appendix (For Online Publication)

Credit Shocks, Lending Relationships and Economic Activity

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Contents

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A Derivation of FOCs

A.1 Households

The Lagrangian of patient households is

$$\mathscr{L}_t = \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \left(\beta^P\right)^t \left[\begin{array}{c} \log\left(C_{i,t}^P - \gamma^P C_{i,t-1}^P\right) - \frac{N_{i,t}^{\eta}}{\eta} + \varsigma \log H_{i,t}^P \\ -\lambda_{i,t}^P \left[\begin{array}{c} C_{i,t}^P + Q_t^H \left(H_{i,t}^P - H_{i,t-1}^P\right) + \int_0^1 D_{ik,t} \mathrm{d}k \\ -W_t N_{i,t} - \int_0^1 \Pi_{ik,t} \mathrm{d}k - R_{t-1}^D \int_0^1 D_{ik,t-1} \mathrm{d}k \end{array} \right] \end{array} \right] \right\} \tag{A.1}$$

The problem yields the following first order conditions (here, I ignore all the i's denoting individual patient households):

$$\frac{\partial \mathscr{L}}{\partial C_t^P} : \frac{1}{C_t^P - \gamma^P C_{t-1}^P} - \beta^P \mathbb{E}_P \frac{\gamma^P}{C_{t+1}^P - \gamma^P C_t^P} = \lambda_t^P \tag{A.2}$$

$$\frac{\partial \mathscr{L}}{\partial D_t} : \beta^P \mathbb{E}_t \lambda_{t+1}^P = \frac{\lambda_t^P}{R_t^D} \tag{A.3}$$

$$\frac{\partial \mathscr{L}}{\partial H_t^P} : \frac{\varsigma}{H_t^P} + \beta^P \mathbb{E}_t \left(\lambda_{t+1}^P Q_{t+1}^H\right) = \lambda_t^P Q_t^H \tag{A.4}$$

$$\frac{\partial \mathscr{L}}{\partial N_t^P} : N_t^{\eta-1} = \lambda_t^P W_t \tag{A.5}$$

A.2 Entrepreneurs

Entrepreneur's optimization problem features two parts. The first part consists of choosing how much to borrow from each individual bank, $l_{jk,t}$ to minimize his total interest rate expenditure. This problem can be framed as

$$\min_{l_{jk,t}} \left[\int_0^1 R_{k,t}^L l_{jk,t} \mathrm{d}k\right] - \chi_t^E \left[x_{j,t} - \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k\right)^{\frac{\xi}{\xi-1}}\right] \tag{A.6}$$

The first order condition for this problem is

$$R_{k,t}^L = -\frac{\xi}{\xi-1} \chi_t^E \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k\right)^{\frac{1}{\xi-1}} \frac{\xi-1}{\xi} \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{-\frac{1}{\xi}} \tag{A.7}$$

This can be rewritten as

$$
\begin{aligned}
R^L_{k,t} &= -\chi^E_t \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k\right)^{\frac{1}{\xi-1}} \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{-\frac{1}{\xi}} \\
R^L_{k,t}\left(l_{jk,t} - \gamma^L s_{k,t-1}\right) &= -\chi^E_t \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k\right)^{\frac{1}{\xi-1}} \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \\
\int_0^1 R^L_{k,t}\left(l_{jk,t} - \gamma^L s_{k,t-1}\right)\mathrm{d}k &= -\chi^E_t \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k\right)^{\frac{1}{\xi-1}} \int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \\
\int_0^1 R^L_{k,t}\left(l_{jk,t} - \gamma^L s_{k,t-1}\right)\mathrm{d}k &= -\chi^E_t \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k\right)^{\frac{\xi}{\xi-1}} \qquad \text{(A.8)}
\end{aligned}
$$

Now, using $\left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k\right)^{\frac{\xi}{\xi-1}} = x_{j,t}$, the previous equation can be written as

$$
x_{j,t} = -\frac{1}{\chi^E_t}\left[\int_0^1 R^L_{k,t}\left(l_{jk,t} - \gamma^L s_{k,t-1}\right)\mathrm{d}k\right] \qquad \ddagger
$$

Define the aggregate lending rate as $R^L_t \equiv \left[\int_0^1 \left(R^L_{k,t}\right)^{1-\xi}\right]^{\frac{1}{1-\xi}}$ and note that at the optimum, the following condition must hold

$$
R^L_t x_{j,t} = \int_0^1 R^L_{k,t}\left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right)\mathrm{d}k
$$

Now, ‡ can be rewritten as

$$
\begin{aligned}
x_{j,t} &= -\frac{1}{\chi^E_t}\left[R^L_t x_{j,t}\right] \\
-\chi^E_t &= R^L_t
\end{aligned}
$$

Inserting this in first order condition (A.8)

$$
\begin{aligned}
R^L_{k,t} &= -\frac{\xi}{\xi-1}\chi^E_t\left(\int_0^1\left(l_{jk,t}-\gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}}\mathrm{d}k\right)^{\frac{1}{\xi-1}}\frac{\xi-1}{\xi}\left(l_{j,t}-\gamma^L s_{k,t-1}\right)^{-\frac{1}{\xi}}\\
R^L_{k,t} &= R^L_t\left(\int_0^1\left(l_{jk,t}-\gamma^L s_{k,t-1}\right)\mathrm{d}k\right)^{\frac{1}{\xi-1}}\left(l_{jk,t}-\gamma^L s_{k,t-1}\right)^{-\frac{1}{\xi}}\\
R^L_{k,t} &= R^L_t\left(x_t\right)^{\frac{1}{\xi}}\left(l_{jk,t}-\gamma^L s_{k,t-1}\right)^{-\frac{1}{\xi}}\\
\left(l_{jk,t}-\gamma^L s_{k,t-1}\right)^{\frac{1}{\xi}} &= \left(x_t\right)^{\frac{1}{\xi}}\frac{R^L_t}{R^L_{k,t}}\\
l_{jk,t} &= \left(\frac{R^L_t}{R^L_{k,t}}\right)^{\xi}x_t+\gamma^L s_{k,t-1}\\
l_{jk,t} &= \left(\frac{R^L_{k,t}}{R^L_t}\right)^{-\xi}x_t+\gamma^L s_{k,t-1}
\end{aligned}
$$

The second part of entrepreneur's optimization problem can be written as

$$
\mathscr{L}_t=\mathbb{E}_t\left\{\sum_{t=0}^{\infty}\left(\beta^E\right)^t\left[\begin{array}{c}\log\left(C^E_{j,t}-\gamma^E C^E_{j,t-1}\right)\\ -\lambda^E_{j,t}\left[\begin{array}{c}C^E_{j,t}+R^L_{k,t-1}\int_0^1 l_{jk,t-1}\mathrm{d}k-Y_{j,t}+W_tN_{j,t}+I_{j,t}\\ +Q^H_t\left(H^E_{j,t}-H^E_{j,t-1}\right)-x_{j,t}\end{array}\right]\\ -\mu^E_{j,t}\left[R^L_{k,t}\int_0^1 l_{jk,t}\mathrm{d}k-\int_0^1\theta\mathrm{d}k\mathbb{E}_t\left(Q^H_{t+1}H^E_{j,t}+Q^K_{t+1}K_{j,t}\right)\right]\\ -\kappa^E_{j,t}\left[K_{j,t}-\left(1-\delta\right)K_{j,t-1}-\left\{1-\frac{\Omega}{2}\left(\frac{I_{j,t}}{I_{j,t-1}}-1\right)^2\right\}I_{j,t}\right]\\ -\epsilon^E_{j,t}\left[x_{j,t}-\left\{\int_0^1\left(l_{jk,t}-\gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}}\mathrm{d}k\right\}^{\frac{\xi}{\xi-1}}\right]\end{array}\right]\right\} \tag{A.9}
$$

where $Y_{j,t}=A_t\left(N_{j,t}\right)^{1-\alpha}\left\{\left(H^E_{j,t-1}\right)^{\phi}\left(K_{j,t-1}\right)^{1-\phi}\right\}^{\alpha}$ may be inserted for $Y_{j,t}$ in the budget constraint. Solving entrepreneur's optmization problem, the first order conditions are (I ignore all

j's here):

$$\frac{\partial \mathcal{L}}{\partial C_t^E} : \frac{1}{C_t^E - \gamma^E C_{t-1}^E} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C_{t+1}^E - \gamma^E C_t^E} = \lambda_t^E \tag{A.10}$$

$$\frac{\partial \mathcal{L}}{\partial x_t} : \lambda_t^E = \epsilon_t^E \tag{A.11}$$

$$\frac{\partial \mathcal{L}}{\partial l_t} : \epsilon_t^E = \beta^E \mathbb{E}_t \lambda_{t+1}^E R_t^L + \mu_t^E R_t^L \tag{A.12}$$

$$\frac{\partial \mathcal{L}}{\partial N_t} : W_t = (1-\alpha) \frac{Y_t}{N_t} \tag{A.13}$$

$$\frac{\partial \mathcal{L}}{\partial H_t^E} : \lambda_t^E Q_t^H = \beta^E \mathbb{E}_t \left\{ \lambda_{t+1}^E \left(Q_{t+1}^H + \alpha\phi \frac{Y_{t+1}}{H_t^E} \right) \right\} + \mu_t^E \theta \mathbb{E}_t Q_{t+1}^H \tag{A.14}$$

$$\frac{\partial \mathcal{L}}{\partial K_t} : \kappa_t^E = \alpha (1-\phi) \beta^E \mathbb{E}_t \left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t} \right) + \beta^E (1-\delta) \mathbb{E}_t \kappa_{t+1}^E + \mu_t^E \theta \mathbb{E}_t Q_{t+1}^K \tag{A.15}$$

$$\frac{\partial \mathcal{L}}{\partial I_t} : \lambda_t^E = \kappa_t^E \left\{ 1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Omega \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) \right\} + \beta^E \Omega \mathbb{E}_t \left\{ \kappa_{t+1}^E \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \right\} \tag{A.16}$$

Using $\lambda_t^E = \epsilon_t^E$ from (A.11), (A.12) becomes

$$\beta^E \mathbb{E}_t \left(\lambda_{t+1}^E R_t^L \right) + \mu_t^E R_t^L = \lambda_t^E \tag{A.17}$$

A.3 Banks

The problem of banks is to choose their lending rate and the total amount of lending. The bank considers deep habits in loan demand. The bank solves the following problem

$$\max_{L_{k,t}, R_{k,t}^L} \Pi_t = R_{k,t-1}^L L_{k,t-1} + \frac{L_{k,t}}{\psi_t} - L_{k,t} - R_{t-1}^D \frac{L_{k,t-1}}{\psi_{t-1}} + \varrho_t^E \left(\int_0^1 \left[\left(\frac{R_t^L}{R_{k,t}^L} \right)^{\xi} x_t + \gamma^L s_{k,t-1} \right] \mathrm{d}j - L_{k,t} \right)$$

The first order condition for $L_{k,t}$ is

$$\left(\frac{1}{\psi_t} - 1 \right) + \mathbb{E}_t q_{t,t+1} R_{k,t}^L - \mathbb{E}_t q_{t,t+1} \frac{R_t^D}{\psi_t} + \gamma^L (1-\rho_s) \mathbb{E}_t \left(q_{t,t+1} \varrho_{t+1}^E \right) - \varrho_t^E = 0$$

Rearranging terms

$$\varrho_t^E = \left(\frac{1}{\psi_t} - 1 \right) + \mathbb{E}_t q_{t,t+1} \left[\left(R_{k,t}^L - \frac{R_t^D}{\psi_t} \right) + \gamma^L (1-\rho_s) \mathbb{E}_t \varrho_{t+1}^E \right] \tag{A.18}$$

The first order condition for $R^L_{k,t}$ is

$$\mathbb{E}_t q_{t,t+1} L_{k,t} + \left(\frac{1}{\psi_t} - 1\right) \xi \varrho^E_t \left(\frac{R^L_t}{R^L_{k,t}}\right)^{\xi-1} x_t \left(\frac{-R^L_t}{\left(R^L_{k,t}\right)^2}\right) + \xi \varrho^E_t \left(\frac{R^L_t}{R^L_{k,t}}\right)^{\xi-1} x_t \left(\frac{-R^L_t}{\left(R^L_{k,t}\right)^2}\right) = 0$$

Moving terms around

$$\mathbb{E}_t q_{t,t+1} L_{k,t} = \left(\frac{1}{\psi_t} - 1\right) \xi \varrho^E_t x_t \left(\frac{R^L_t}{R^L_{k,t}}\right)^{\xi-1} \left(\frac{R^L_t}{\left(R^L_{k,t}\right)^2}\right) + \xi \varrho^E_t x_t \left(\frac{R^L_t}{R^L_{k,t}}\right)^{\xi-1} \left(\frac{R^L_t}{\left(R^L_{k,t}\right)^2}\right) \quad \text{(A.19)}$$

In a symmetric equilibrium all banks have the same lending rate $R^L_{k,t} = R^L_t, \forall k$ and consequently lend the same amount $L_{k,t} = L_t, \forall k$. Bank's first order condition in this case can be rewritten as

$$\varrho^E_t = \left(\frac{1}{\psi_t} - 1\right) + \mathbb{E}_t q_{t,t+1} \left[\left(R^L_t - \frac{R^D_t}{\psi_t}\right) + \gamma^L (1 - \rho_s) \mathbb{E}_t \varrho^E_{t+1}\right] \quad \text{(A.20)}$$

$$\frac{1}{\psi_t} \frac{\xi \varrho^E_t x_t}{R^L_t} = \mathbb{E}_t q_{t,t+1} L_t \quad \text{(A.21)}$$

where I have imposed $L_t = l_t$ in a symmetric equilibrium.

B List of Equations

B.1 Households

$$\frac{1}{C^P_t - \gamma^P C^P_{t-1}} - \beta^P \mathbb{E}_t \frac{\gamma^P}{C^P_{t+1} - \gamma^P C^P_t} = \lambda^P_t \quad \text{(B.1)}$$

$$\beta^P \mathbb{E}_t \lambda^P_{t+1} = \frac{\lambda^P_t}{R^D_t} \quad \text{(B.2)}$$

$$\frac{\varsigma}{H^P_t} + \beta^P \mathbb{E}_t \left(\lambda^P_{t+1} Q^H_{t+1}\right) = \lambda^P_t Q^H_t \quad \text{(B.3)}$$

$$N^{\eta-1}_t = \lambda^P_t W_t \quad \text{(B.4)}$$

B.2 Entrepreneurs

$$\frac{1}{C^E_t - \gamma^E C^E_{t-1}} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C^E_{t+1} - \gamma^E C^E_t} = \lambda^E_t \quad \text{(B.5)}$$

$$\beta^E \mathbb{E}_t \left(\lambda^E_{t+1} R^L_t\right) + \mu^E_t R^L_t = \lambda^E_t \quad \text{(B.6)}$$

$$W_t = (1-\alpha)\frac{Y_t}{N_t} \tag{B.7}$$

$$\lambda_t^E Q_t^H = \beta^E \mathbb{E}_t \left\{ \lambda_{t+1}^E \left(Q_{t+1}^H + \alpha\phi \frac{Y_{t+1}}{H_t^E} \right) \right\} + \mu_t^E \theta \mathbb{E}_t Q_{t+1}^H \tag{B.8}$$

$$\kappa_t^E = \alpha(1-\phi)\beta^E \mathbb{E}_t \left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t} \right) + \beta^E (1-\delta)\mathbb{E}_t \kappa_{t+1}^E + \mu_t^E \theta \mathbb{E}_t Q_{t+1}^K \tag{B.9}$$

$$\lambda_t^E = \kappa_t^E \left\{ 1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Omega \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) \right\} + \beta^E \Omega \mathbb{E}_t \left\{ \kappa_{t+1}^E \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \right\} \tag{B.10}$$

$$s_t = \rho_s s_{t-1} + (1-\rho_s) l_t \tag{B.11}$$

$$x_t = \left(l_t - \gamma^L s_{t-1} \right) \tag{B.12}$$

$$L_t = l_t \tag{B.13}$$

$$C_t^E + R_{t-1}^L l_{t-1} = Y_t - W_t N_t - I_t - Q_t \left(H_t^E - H_{t-1}^E \right) + x_t \tag{B.14}$$

$$l_t = \frac{\theta a_t}{R_t^L} \tag{B.15}$$

$$a_t = \mathbb{E}_t \left(Q_{t+1}^H H_t^E + Q_{t+1}^K K_t \right) \tag{B.16}$$

$$\kappa_t^E = \lambda_t^E Q_t^K \tag{B.17}$$

B.3 Banks

$$\varrho_t^E = \left(\frac{1}{\psi_t} - 1 \right) + \mathbb{E}_t q_{t,t+1} \left[\left(R_t^L - \frac{R_t^D}{\psi_t} \right) + \gamma^L (1-\rho_s) \mathbb{E}_t \varrho_{t+1}^E \right] \tag{B.18}$$

$$\frac{1}{\psi_t} \xi \varrho_t^E x_t \frac{1}{R_t^L} = \mathbb{E}_t q_{t,t+1} L_t \tag{B.19}$$

$$\Pi_t = R_{t-1}^L L_{t-1} + D_t - L_t - R_{t-1}^D D_{t-1} \tag{B.20}$$

$$L_t = \psi_t D_t \tag{B.21}$$

$$q_{t,t+1} = \beta^P \mathbb{E}_t \frac{\lambda_{t+1}^P}{\lambda_t^P} \tag{B.22}$$

B.4 Market Clearing and Resource Constraints

$$C_t^P + C_t^E + I_t = Y_t \tag{B.23}$$

$$H_t^P + H_t^E = H \tag{B.24}$$

$$Y_t = A_t (N_t)^{1-\alpha} \left\{ \left(H_{t-1}^E\right)^{\phi} (K_{t-1})^{1-\phi} \right\}^{\alpha} \tag{B.25}$$

$$K_t = (1-\delta) K_{t-1} + \left\{ 1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right\} I_t \tag{B.26}$$

C Steady State Conditions

All $i's$, $j's$ and $k's$ denoting individual household, entrepreneur and bank respectively are ignored. From household's FOC with respect to consumption (B.1) and labor (B.4), I have

$$\frac{1-\beta^P\gamma^P}{\left(1-\gamma^P\right)C^P} = \lambda^P \tag{C.1}$$

and

$$N^{\eta-1} = \lambda^P W \tag{C.2}$$

respectively. Household's FOC with respect to deposit (B.2) yields the steady-state gross interest rate

$$R^D = \frac{1}{\beta^P} \tag{C.3}$$

underscoring that the time preference of most patient individual determines the steady-state rate of interest. From (B.3), I obtain

$$\begin{aligned} \frac{\varsigma}{H^P} + \beta^P\lambda^P Q^H &= \lambda^P Q^H \\ \Rightarrow Q^H H^P &= \frac{\varsigma}{\lambda^P\left(1-\beta^P\right)} \\ \Rightarrow H^P &= \frac{\varsigma}{Q^H\lambda^P\left(1-\beta^P\right)} \end{aligned} \tag{C.4}$$

I next turn to entrepreneurs. Their consumption FOC (B.5) yields

$$\frac{1-\beta^E\gamma^E}{\left(1-\gamma^E\right)C^E} = \lambda^E \tag{C.5}$$

Entrepreneur's FOC with respect to loans (B.6) gives

$$\begin{aligned} \beta^E\lambda^E R^L + \mu^E R^L &= \lambda^E \\ \Rightarrow \mu^E &= \frac{\lambda^E\left(1-\beta^E R^L\right)}{R^L} \end{aligned} \tag{C.6}$$

The borrowing constraint for entrepreneurs binds only if μ^E is positive. This implies that β^E must be less than R^L. In the baseline calibration, β^E is set to 0.95 whereas the steady state value of R^L is 1.0219 which implies that β^E must be less than 0.9786 which is indeed the case. The production function is

$$Y = A(N)^{1-\alpha}\left[\left(H^E\right)^{\phi}(K)^{1-\phi}\right]^{\alpha} \tag{C.7}$$

From firm's labor choice for househods (B.7),

$$W = (1-\alpha)\frac{Y}{N} \tag{C.8}$$

From entrepreneur's FOC with respect to housing (B.8), I have

$$\begin{aligned} \lambda^E Q^H &= \beta^E \lambda^E \left(Q^H + \alpha\phi\frac{Y}{H^E}\right) + \mu^E \theta Q^H \\ \Rightarrow \frac{Q^H H^E}{Y} &= \frac{\beta^E \alpha\phi R^L}{\left(1-\beta^E\right)R^L - \theta\left(1-\beta^E R^L\right)} \end{aligned} \tag{C.9}$$

From aggregate law of motion for capital (B.26)

$$\begin{aligned} K &= (1-\delta)K + \left[1 - \frac{\Omega}{2}\left(\frac{I}{I} - 1\right)\right] I \\ &\Rightarrow I = \delta K \end{aligned} \tag{C.10}$$

I have the following steady-state resource constraints

$$Y = C^P + C^E + I \tag{C.11}$$

$$H = H^P + H^E \tag{C.12}$$

$$L = \psi D \tag{C.13}$$

Also, I have the following steady-state version of agents' budget constraints (one of them is redundant because of Walras' Law)

$$C^P = WN - (R^D - 1)D + \Pi \tag{C.14}$$

$$C^E = Y - R^L l - WN - I - x \tag{C.15}$$

So the steady state is characterized by the vector

$$\left[Y, C^P, C^E, I, H^P, H^E, K, N, L, D, Q^H, Q^K, R^D, R^L, W, \lambda^P, \lambda^E, \mu^E\right]$$

From entrepreneur's optimal choice of capital (B.9), I have

$$\kappa_t^E = \alpha(1-\alpha)\beta^E \mathbb{E}_t\left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t}\right) + \beta^E(1-\delta)\mathbb{E}_t\kappa_{t+1}^E + \mu_t^E\theta_t\mathbb{E}_t Q_{t+1}^K$$
$$\Rightarrow \frac{\kappa^E}{\lambda^E}\left(1-(1-\delta)\beta^E\right) = \alpha(1-\phi)\beta^E\frac{Y}{K} + \frac{\left(1-\beta^E R^L\right)}{R^L}\theta Q^K \tag{C.16}$$

Entrepreneur's optimal choice of investment (B.10) yields

$$\lambda_t^E(j) = \kappa_t^E(j)\left[1 - \frac{\Omega}{2}\left(\frac{I_t(j)}{I_t(j-1)} - 1\right)^2 - \Omega\frac{I_t(j)}{I_t(j-1)}\left(\frac{I_t(j)}{I_{t-1}(j)} - 1\right)\right]$$
$$+ \beta^E\Omega\mathbb{E}_t\left[\kappa_{t+1}^E(j)\left(\frac{I_{t+1}(j)}{I_t(j)}\right)^2\left(\frac{I_{t+1}(j)}{I_t(j)} - 1\right)\right]$$
$$\Rightarrow \lambda^E = \kappa^E \tag{C.17}$$

Combining this with steady state version of

$$\kappa^E = \lambda^E Q^K \tag{C.18}$$

I obtain $Q^K = 1$ in the steady state. Plugging this into (C.16), I obtain the expression for capital-to-output ratio

$$\frac{\kappa^E}{\lambda^E}\left(1-(1-\delta)\beta^E\right) = \alpha(1-\phi)\beta^E\frac{Y}{K} + \frac{\left(1-\beta^E R^L\right)}{R^L}\theta Q^K$$
$$\Rightarrow \frac{K}{Y} = \frac{\alpha(1-\phi)R^L\beta^E}{R^L\left(1-(1-\delta)\beta^E\right) - \theta\left(1-\beta^E R^L\right)} \tag{C.19}$$

Next, combining (B.15) and (B.16) yields

$$l = \frac{\theta}{R^L}\left[Q^H H^E + Q^K K\right] \tag{C.20}$$

Dividing by Y, the above expression becomes

$$\frac{l}{Y} = \frac{\theta}{R^L}\left[\frac{Q^H H^E}{Y} + \frac{Q^K K}{Y}\right]$$

Plugging in the values of $\frac{Q^H H^E}{Y}$ and $\frac{K}{Y}$ and using that $Q^K = 1$, I have

$$\frac{l}{Y} = \alpha\theta\beta^E \left[\frac{\phi}{R^L\left(1-\beta^E\right) - \theta\left(1-\beta^E R^L\right)} + \frac{(1-\phi)}{R^L\left(1-(1-\delta)\beta^E\right) - \theta\left(1-\beta^E R^L\right)} \right] \tag{C.21}$$

From entrepreneur's budget constraint (B.14)

$$C^E + R^L l = Y - WN - I + x \tag{C.22}$$

Rewriting this in ratios to output

$$\begin{aligned} \frac{C^E}{Y} + \frac{R^L l}{Y} &= 1 - \frac{WN}{Y} - \frac{I}{Y} + \frac{x}{Y} \\ \Rightarrow \frac{C^E}{Y} &= \alpha - \delta\frac{K}{Y} + \left(1 - \gamma^L - R^L\right)\frac{l}{Y} \end{aligned} \tag{C.23}$$

Dividing (C.4) by Y and then dividing it again by (C.9) gives

$$\begin{aligned} \frac{\frac{Q^H H^P}{Y}}{\frac{Q^H H^E}{Y}} &= \frac{\frac{\varsigma}{Y\lambda^P\left(1-\beta^P\right)}}{\frac{\beta^E \alpha\phi R^L}{\left(1-\beta^P\right)R^L - \theta\left(1-\beta^E R^L\right)}} \\ \Rightarrow \frac{H^P}{H^E} &= \frac{\varsigma}{Y\frac{1-\beta^P\gamma^P}{\left(1-\gamma^P\right)C^P}\left(1-\beta^P\right)} \frac{\left(1-\beta^E\right)R^L - \theta\left(1-\beta^E R^L\right)}{\beta^E\alpha\phi R^L} \\ \Rightarrow \frac{H^P}{H - H^P} &= \frac{\varsigma\left(1-\gamma^P\right)}{\left(1-\beta^P\right)\left(1-\beta^P\gamma^P\right)} \frac{\left(1-\beta^P\right)R^L - \theta\left(1-\beta^E R^L\right)}{\beta^E\alpha\phi R^L}\frac{C^P}{Y} \end{aligned} \tag{C.24}$$

From entrepreneur's stock of habits for loans (B.11)

$$\begin{aligned} s_t &= \rho_s s_{t-1} + \left(1-\rho_s\right) l_t \\ s &= l \end{aligned} \tag{C.25}$$

Entrepreneur's effective demand for loans (B.12) gives

$$\begin{aligned} x_t &= \left(l_t - \gamma^L s_{t-1}\right) \\ \Rightarrow x &= \left(1-\gamma^L\right) l \end{aligned} \tag{C.26}$$

Total loans of entrepreneurs (B.13)

$$L = l \tag{C.27}$$

From bank's balance sheet condition (B.21)

$$L = \psi D \tag{C.28}$$

Steady state version of stochastic discount factor (B.22) reads

$$q = \beta^P \tag{C.29}$$

The steady-state version of bank's first order condition (B.18) with respect to loans reads

$$\varrho^E = \left(\frac{1}{\psi} - 1\right) + \beta^P \left[R^L - \frac{R^D}{\psi} + \gamma^L (1 - \rho_s) \varrho^E\right]$$

which can be simplified to yield

$$\varrho^E = \frac{\left(\frac{1}{\psi} - 1\right) + \beta^P \left(R^L - \frac{R^D}{\psi}\right)}{1 - \beta^P \gamma^L (1 - \rho_s)} \tag{C.30}$$

The steady-state version of bank's second first order condition with respect to lending rate (B.19) writes

$$\frac{1}{\psi} \xi \varrho^E x \frac{1}{R^L} = \beta^P L$$

Steady-state version of aggregate resource constraint (B.23) is

$$\begin{aligned} C^P + C^E + I &= Y \\ \Rightarrow \frac{C^P}{Y} &= 1 - \frac{C^E}{Y} - \delta \frac{K}{Y} \end{aligned} \tag{C.31}$$

Combining (C.1), (C.2) and (C.8) gives steady-state equilibrium condition for households

$$\begin{aligned} N^{\eta-1} &= \lambda^P W \\ \Rightarrow N^{\eta-1} &= \frac{1 - \beta^P \gamma^P}{(1 - \gamma^P) C^P} (1 - \alpha) \frac{Y}{N} \\ \Rightarrow N &= \left[\frac{\left(1 - \beta^P \gamma^P\right)(1 - \alpha)}{(1 - \gamma^P)} \left(\frac{C^P}{Y}\right)^{-1}\right]^{\frac{1}{\eta}} \end{aligned} \tag{C.32}$$

From (B.25), steady state output is

$$\begin{aligned}
Y &= A\left(N\right)^{1-\alpha}\left[\left(H^{E}\right)^{\phi}\left(K\right)^{1-\phi}\right]^{\alpha} \\
Y^{1-\alpha} &= A\left(N\right)^{1-\alpha}\left[\left(\frac{H^{E}}{Y}\right)^{\phi}\left(\frac{K}{Y}\right)^{1-\phi}\right]^{\alpha} \\
Y^{1-\alpha} &= A\left(N\right)^{1-\alpha}\left[\left(\frac{H^{E}}{Y}\right)^{\phi}\left(\frac{\alpha\left(1-\phi\right)R^{L}\beta^{E}}{R^{L}\left(1-\left(1-\delta\right)\beta^{E}\right)-\theta\left(1-\beta^{E}R^{L}\right)}\right)^{1-\phi}\right]^{\alpha} \qquad \text{(C.33)}
\end{aligned}$$

From (C.4)

$$Q^{H}=\frac{\varsigma}{H^{P}\lambda^{P}\left(1-\beta^{P}\right)} \qquad \text{(C.34)}$$

D System of Loglinear Equations

The system of equations log-linearized around their steady state is as below:

D.1 Optimality Conditions of Households

(A.2), (A.3) and (A.5) become

$$\beta^{P}\gamma^{P}\mathbb{E}_{t}\widehat{C}_{t+1}^{P}-\left(1+\left(\gamma^{P}\right)^{2}\beta^{P}\right)\widehat{C}_{t}^{P}+\gamma^{P}\widehat{C}_{t-1}^{P}=\left(1-\beta^{P}\gamma^{P}\right)\left(1-\gamma^{P}\right)\widehat{\lambda}^{P} \qquad \text{(D.1)}$$

$$\mathbb{E}_{t}\widehat{\lambda}_{t+1}^{P}=\widehat{\lambda}_{t}^{P}-\widehat{R}_{t}^{D} \qquad \text{(D.2)}$$

$$\left(\eta-1\right)\widehat{N}_{t}=\widehat{\lambda}_{t}^{P}+\widehat{W}_{t} \qquad \text{(D.3)}$$

Log-linearization of (A.4) gives

$$\beta^{P}\mathbb{E}_{t}\left[\widehat{\lambda}_{t+1}^{P}+\widehat{Q}_{t+1}^{H}+\widehat{H}_{t}^{P}\right]=\widehat{\lambda}_{t}^{P}+\widehat{Q}_{t}^{H}+\widehat{H}_{t}^{P} \qquad \text{(D.4)}$$

D.2 Optimality Conditions of Entrepreneurs

From (B.5) and (B.6), I have

$$\beta^{E}\gamma^{E}\mathbb{E}_{t}\widehat{C}_{t+1}^{E}-\left(1+\left(\gamma^{E}\right)^{2}\beta^{E}\right)\widehat{C}_{t}^{E}+\gamma^{E}\widehat{C}_{t-1}^{E}=\left(1-\beta^{E}\gamma^{E}\right)\left(1-\gamma^{E}\right)\widehat{\lambda}_{t}^{E} \qquad \text{(D.5)}$$

and

$$\widehat{\lambda}_t^E = \widehat{R}_t^L + \beta^E R^L \mathbb{E}_t \widehat{\lambda}_{t+1}^E + \left(1 - \beta^E R^L\right) \widehat{\mu}_t^E \tag{D.6}$$

(B.7) yields

$$\widehat{W}_t = \widehat{Y}_t - \widehat{N}_t \tag{D.7}$$

From (B.8), I derive

$$\begin{aligned}\left(\widehat{\lambda}_t^E + \widehat{Q}_t^H\right) = \beta^E \mathbb{E}_t\big(\widehat{\lambda}_{t+1}^E + \widehat{Q}_{t+1}^H\big) + \left(\frac{1}{R^L} - \beta^E\right)\theta \mathbb{E}_t\big(\widehat{\mu}_t^E + \widehat{\theta}_t + \widehat{Q}_{t+1}^H\big) \\ + \left[\left(1 - \beta^E\right) - \theta\left(\frac{1}{R^L} - \beta^E\right)\right]\mathbb{E}_t\big[\widehat{\lambda}_{t+1}^E + \widehat{Y}_{t+1} - \widehat{H}_t^E\big]\end{aligned} \tag{D.8}$$

(B.9) becomes

$$\begin{aligned}\widehat{Q}_t^K = \big[1 - \beta^E\big(1 - \delta\big) - \theta\big(\frac{1}{R^L} - \beta^E\big)\big]\mathbb{E}_t\big[\widehat{\lambda}_{t+1}^E - \lambda_t^E + \widehat{Y}_{t+1} - K_t\big] \\ + \beta^E\big(1 - \delta\big)\mathbb{E}_t\big(\widehat{Q}_{t+1}^K + \widehat{\lambda}_{t+1}^E - \widehat{\lambda}_t^E\big) + \big(1 - \beta^E R^L\big)\frac{1}{R^L}\theta \mathbb{E}_t\left[\widehat{\mu}_t^E - \widehat{\lambda}_t^E + \widehat{\theta}_t + \widehat{Q}_{t+1}^K\right]\end{aligned} \tag{D.9}$$

(B.10) is approximated as

$$\widehat{Q}_t^K = \left(1 + \beta^E\right)\Omega \widehat{I}_t - \beta^E \Omega \mathbb{E}_t \widehat{I}_{t+1} - \Omega \widehat{I}_{t-1} \tag{D.10}$$

From (B.11) and (B.13), I get

$$\widehat{s}_t = \rho_s \widehat{s}_{t-1} + (1 - \rho_s)\,\widehat{l}_t \tag{D.11}$$

and

$$\widehat{x}_t = \frac{\widehat{l}_t}{1 - \gamma^L} - \frac{\gamma^L \widehat{s}_{t-1}}{1 - \gamma^L} \tag{D.12}$$

(B.14) becomes

$$C^E \widehat{C}_t^E + R^L l\left(\widehat{R}_{t-1}^L + \widehat{l}_{t-1}\right) = Y\widehat{Y}_t - WN\left(\widehat{W}_t + \widehat{N}_t\right) - I\widehat{I}_t - Q^H H^E\left(\widehat{H}_t^E - \widehat{H}_{t-1}^E\right) + x\widehat{x}_t \tag{D.13}$$

(B.15) gives

$$\widehat{l}_t = \widehat{\theta}_t + \widehat{a}_t - \widehat{R}_t^L \tag{D.14}$$

(B.16) yields

$$\widehat{a}_t = \frac{Q^H H^E}{Q^H H^E + Q^K K} \mathbb{E}_t \left(\widehat{Q}^H_{t+1} + \widehat{H}^E_t \right) + \frac{Q^K K}{Q^H H^E + Q^K K} \mathbb{E}_t \left(\widehat{Q}^K_{t+1} + \widehat{K}_t \right) \tag{D.15}$$

Linearized versions of (B.17) is

$$\widehat{\kappa}^E_t = \widehat{\lambda}^E_t + \widehat{Q}^K_t \tag{D.16}$$

D.3 Optimality Conditions of Banks

From (B.18), I obtain

$$\begin{aligned} \frac{\psi \varrho^E}{\beta^P} \widehat{\psi}_t \widehat{\varrho}^E_t - \psi \varrho^E \gamma^L (1-\rho_s) \left(\mathbb{E}_t \widehat{\varrho}^E_{t+1} + \widehat{\psi}_t \right) =& - \psi \widehat{\psi}_t + \left[\psi R^L - R^D + \psi \varrho^E \gamma^L (1-\rho_s) \right] \mathbb{E}_t \widehat{q}_{t,t+1} \\ &+ \psi R^L \left(\widehat{R}^L_t + \widehat{\psi}_t \right) - R^D \widehat{R}^D_t \end{aligned} \tag{D.17}$$

Log-linearization of (B.19) yields

$$\xi \varrho^E x \left(\widehat{\varrho}^E_t + \widehat{x}_t \right) = \beta^P \psi R^L L \left(\widehat{\psi}_t + \widehat{R}^L_t + \widehat{L}_t + \mathbb{E}_t \widehat{q}_{t,t+1} \right) \tag{D.18}$$

From (B.21), I get

$$L \widehat{L}_t = \psi \widehat{\psi}_t + D \widehat{D}_t \tag{D.19}$$

Log-linearization of (B.22) is

$$\widehat{q}_{t,t+1} = \widehat{\lambda}^P_{t+1} - \widehat{\lambda}^P_t \tag{D.20}$$

D.4 Market Clearing and Resource Constraints

From (B.13), I obtain

$$\widehat{L}_t = \widehat{l}_t \tag{D.21}$$

(B.23) and (B.24) yield

$$\widehat{Y}_t = \frac{C^P}{Y} \widehat{C}^P_t + \frac{C^E}{Y} \widehat{C}^E_t + \frac{I}{Y} \widehat{I}_t \tag{D.22}$$

and

$$H^P \widehat{H}^P_t + H^E \widehat{H}^E_t = 0 \tag{D.23}$$

From (B.25), I have

$$\widehat{Y}_t = \widehat{A}_t + (1-\alpha)\,\widehat{N}_t + \alpha\phi\widehat{H}^E_{t-1} + \alpha\,(1-\phi)\,\widehat{K}_{t-1} \tag{D.24}$$

(B.26) gives

$$\widehat{K}_t = (1-\delta)\,\widehat{K}_{t-1} + \delta\widehat{I}_t \tag{D.25}$$