Twin Debts, Endogenous Lending Standards and Macroeconomic Fluctuations

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Abstract

This paper presents a model where both households and firms form lending relationships with banks. The banks in this model compete on both interest rates and collateral and the economy features an endogenously evolving credit standard. The results in the paper show that lending relationships amplify macroeconomic fluctuations via-a-vis the case where no lending relationships exist.

Keywords: Lending Standards, Deep Habits in Banking, Macroeconomic Fluctuations

JEL Classification: E32, E44

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1 Introduction

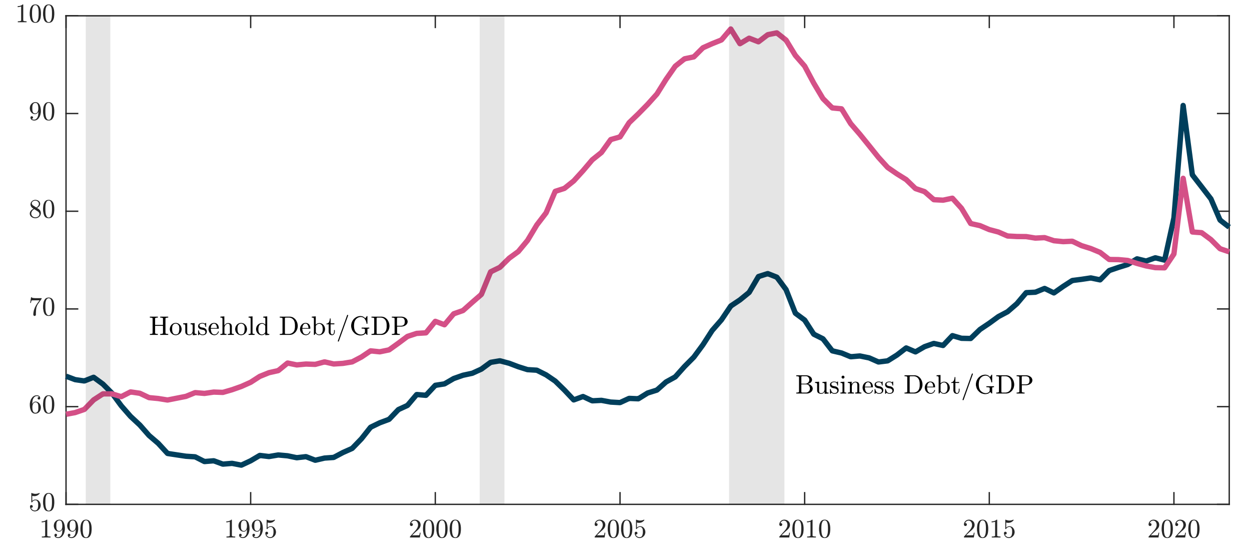
The contribution of this paper is to show that systematic variation in credit markets can explain significant macroeconomic fluctuations. To this aim, this paper presents a model in which both households and firms borrow from a continuum of banks. Over time, these borrowers develop endogenously-persistent credit relationships[1] with their lenders. Banks in this model compete on both interest rates and collateral requirements. Competition on collateral requirements leads to an endogenously-evolving lending standrad in the economy. The results in this paper show that these credit relationships amplify the effects of various macroeconomic shocks. All the shocks I consider in this paper – a technology shock, a housing demand shock and a labor supply shock – show remarkably higher impact when the economy features lending relationships versus the case when it doesn't. This paper contributes to the literature by demonstrating that lender-borrower credit relationships can play important role in amplifying macroeconomic fluctuations.

This work focuses on addressing the question of what drives aggregate fluctuations in a model where both firms and households have lending relationsips with banks and how much they can borrow is limited by the value of their collateral. Existing work, for example Ravn (2016), consider this question in a model where borrowing is done by firms. These works ignore the important role of significant amount of borrowing by households and the fact that defualts by them can have significant impact on banks which can ripple out to the wider macroeconomy.

A look at Figure 1 shows that for almost entire period of 1990 to 2020, household debt has been higher than business debt. This implies that it is importat to include household debt in any model that seeks to explain macroeconomic fluctuations using lending. At its simplest, it suggests that models that leave out household debt and aim to explain business cycle fluctuations by including business debt in their models might be missing important dynamics and they might be underestimating the true magnitude of including debt in their models. Household debt exceeded corporate debt in 1990 and accounted for as much as 98% of US GDP in 2009. Though households borrow for a variety of reasons such as vehicle loans, card loans and education loans, a vast majority of household borrowing is on account of residential mortgages. This paper argues that deep habits in banking can explain significant business cycle variations in the US economy. While making this point, this paper emphasizes the importance of household loans and offers a

[1] I will be using the terms 'credit relationships' and 'lending relationships' synonymously throughout this paper.

Figure 1: Household Debt to GDP and (non-financial) Businesss Debt to GDP ratio for US.



Note: Household debt includes debt securities and loans of households and nonprofit organizations. Corporate debt includes debt securities and loans of nonfinancial businesses. Data from FRB San Francisco. Shaded areas refer to NBER recession dates.

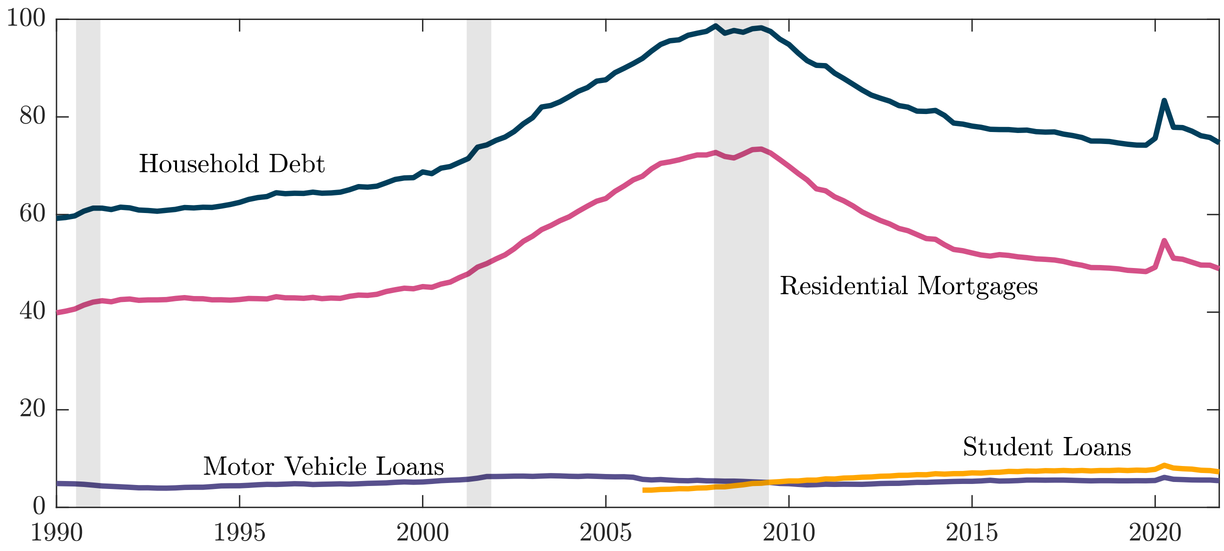
framework that can include both household and corporate debt.

Majority of the household debt is made of residential mortgages. A small part comes from motor vehicle loans and student loans. Figure 2 shows how most of the changes in household debt over the years has mostly come from changes in residential ortgages.

The large amount of household debt in the US also ties with in the fact that household indebtedness is not limited to a few US states and it's rather widespread. Figure 3 shows that average debt-to-income (DTI) ratio for the US states over the period 1999-2021. It's clear that most states feature a DTI ratio in excess of unity. This fact has important ramifications for any model that includes and seeks to explain its macroeconomic implications.

There is widespread consensus that the financial crisis of 2007-08 was caused, at least in part, by defaults on loans by subprime borrowers. These were households with weak credit scores who bought houses on loans which they later defaulted on when they could not pay the loans. A comprehensive look at the effects of endogenous lending standards on aggregate fluctuations, therefore, requires that defaults by both households and firms are cosnidered and taken into account in models which are used to gauge their impact. While there is abundant literature on defaults, there is surprisingly scarce work on deep-habit borrowings by both houshods and firms, defualts by both and their collective impact on aggregate variables. This paper represents progress towards this goal – it features a model in which borrowing by both househods

Figure 2: Household Debt Decomposition for the US.

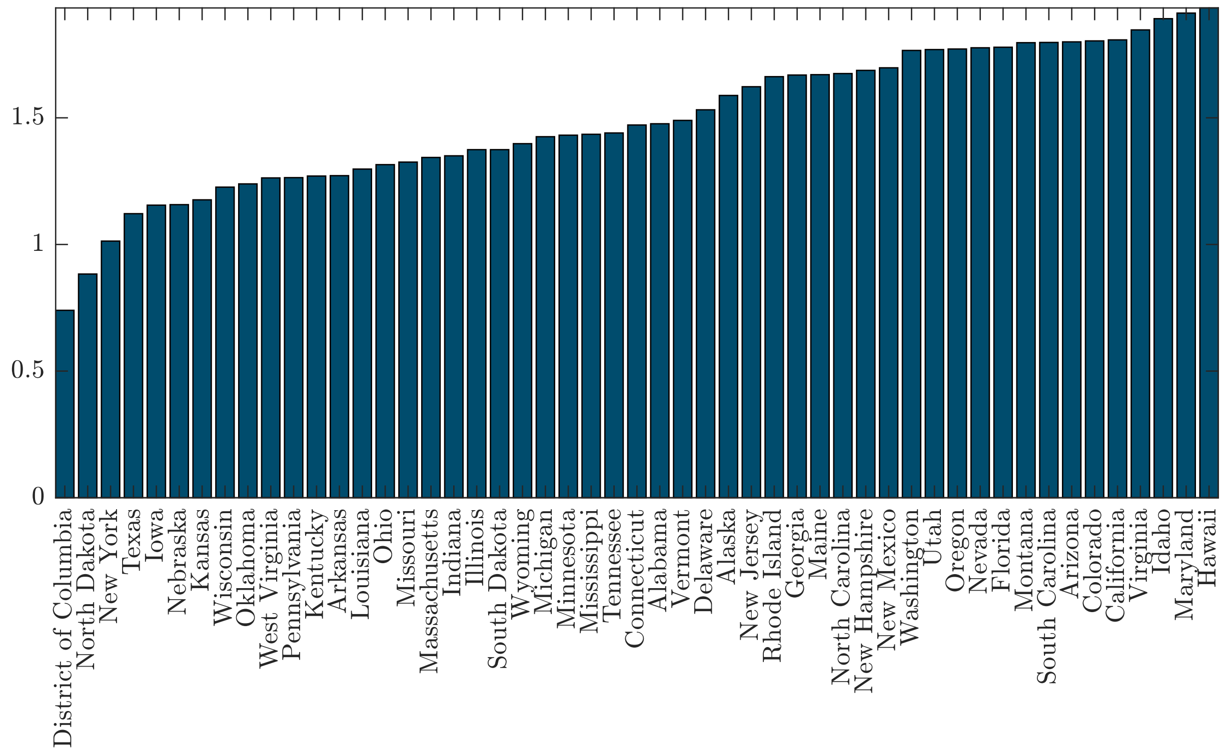


Note: Residential mortgages refer to one-to-four-family residential mortgages. Motor vehicle loans refer to motor vehicle loans owned and securitized. Student loans refer to student loans owned and securitized. Data from FRB San Francisco. Shaded areas refer to NBER recession dates.

and entrprenesures who own firms display deep-habit borrowings. The credit standards in the economy evolves endogeneously which means that during economic upturns, banks relax their credit standards which they tighten in economic slowdown. The model features defaults by both households and firms in the equilibrium which then effects aggregate fluctuations.

The important insight that emerges from this paper is that endogeneous variations in credit standards over business cycles can explain a sizable portion of economic fluctuations. The exisiting literature has not focused on the joint role of household and business loans in a model environment of endogeneously varying credit standards and this paper fills this gap. I present a model which features lending by both households and businesses. The households in this paper are of two types – one that supplies labor, consumes and saves and the other which also supplies labor and cosumes but differently from the first type of households, borrows from banks. I call the first households patient while the second types of households are labled impatient. Becuase of their lower discount factor, impatient households end up borrowing in the equilibrium. The model contains another class of agents called entrepreneurs who are also impatient and they too borrow from banks. Banks in this model economy raise funding in the form of despoits from patient households. These deposits are the only source of funding for the banks.

Figure 3: Debt-to-Income Ratio



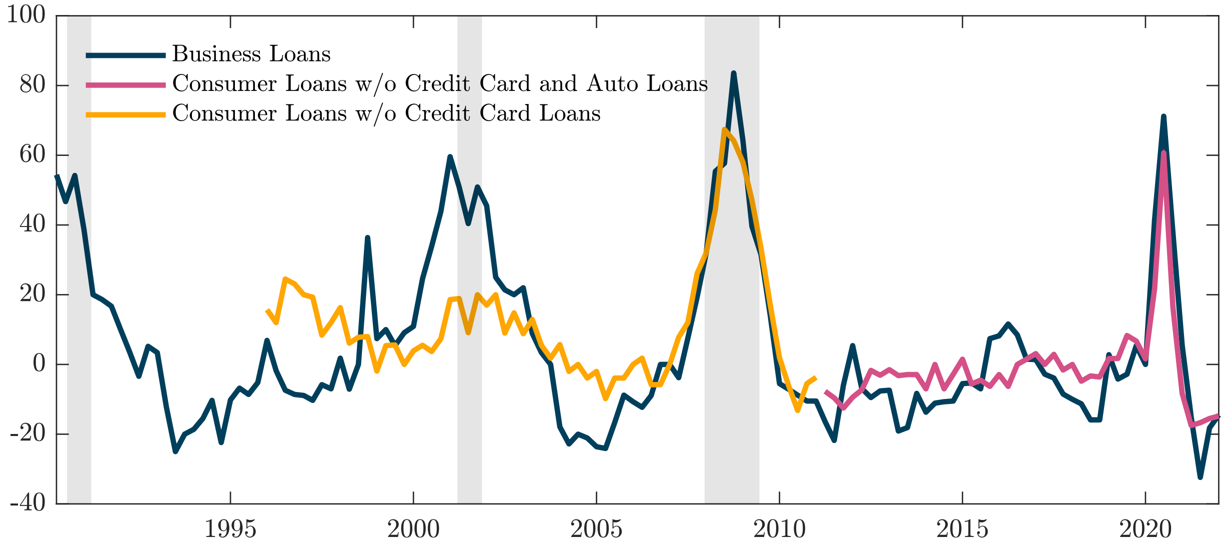
Note: US states ordered by the average household debt-to-income ratio over 1999 to 2021. Data from FRBNY Consumer Credit Panel.

2 Related Work

There is a significant body of work on lending standards and macroeconomic outcomes. Of late, some of the papers that have employed variations in credit standards to explain macroeconomic outcomes and aggregate dynamics include work of Ravn (2016) and Gete (2018), among others. These papers feature financial frictions and a mechanism to characterise changes in lending standards over the business cycles. The common finding from this literature is that there is sizable variation in credit standards over the business cycles and these variations matter because they have important effects on investment, output and consumption.

There is a sprawling literature that documents systematic fluctuation in credit standards. Papers describing changes in collateral requirements or spread between deposit and lending rates include Rajan (1994), Ruckes (2004) and Dell'Ariccia and Marquez (2006). (Petersen and Rajan, 1994, 1995) are examples of Empirical studies of lending relationships. Aliaga-Díaz and Olivero (2010) discuss lending relationships modelled using deep habits in loan demand from individual banks á la the deep habits framework developed by Ravn, Schmitt-Grohé, and Uribe (2006). Justification for deep habits model – inforamtion asymmetry between lenders and borrowers

Figure 4: Net Percentage of Banks Tightening Standards.



Note: Business loans refer to commercial and industry loans to large and middle-market firms. Data series for conusmer loans w/o credit card loans began in 1996 and was discontinued in 2011. Data series for consumer loans w/o credit card loans and auto loans began in 2011. Data from Senior Loan Officer Opinion Survey on Bank Lending Practices administered by the Federal Reserve. Shaded areas refer to NBER recession dates.

Sharpe (1990), Kim, Kliger, and Vale (2003). Propensity of borrowers to switch from their lenders negatively related with duration of relationship (Chakravarty, Feinberg, and Rhee, 2004) and perceived reliability and resposiveness. Competition on lending rates has been studied by Gerali, Neri, Sessa, and Signoretti (2010) and competition on collateral requirements by Ravn (2016). 80% of US small business loans collaterized (Avery, Bostic, and Samolyk, 1998). Non-price competition in banking due to agency problems with price competition (Stiglitz and Weiss, 1981). (Bestor, 1985) presents a model in which banks compete both on lending rates and collateral requirements. Borrowers with long banking relationships have been found to be less likely to pledge collateral (Berger and Udell, 1995). In Ravn (2016) credit standards act as additonal accelerator to financial accelerator of Kiyotaki and Moore (1997). This generates amplification of technology shocks, an effect usually not produced by financial frictions (Kocherlakota, 2000; Liu, Wang, and Zha, 2013). Jensen, Ravn, and Santoro (2018), however, demosntrates that in a model with two types of credit constrained agents, strategic complementarities between their repective collateral constraints can create quantitatively relevant amplification of technology shocks. Aliaga-Díaz and Olivero (2010) show deep habits in banking may generate countercyclical spreads between lending and deposit rates as observed in data. Aksoy, Basso, and Coto-Martinez (2013) report a small effect of lending relationships on amplification of output fluctuations. Airaudo, Olivero et al. (2014) embed deep habits in banking with cost channel of monetary policy. Melina and Villa (2014) show that with deep habits in banking countercyclical movements in interest rates lead to increased government spending multiplier. Melina and Villa (2018) present a DSGE model with banking relationships. Dell'Ariccia and Marquez (2006) show that during booms, banks lower their collateral requirements to attract more borrrowers. Ruckes (2004) argues that banks have to offer more attractive borrowing terms during booms to their customers since there is increased competition amongst banks for them during an economic upturn. short term concerns relative to other banks (Rajan, 1994). Other papers Berlin and Butler (2002), Hainz, Weill, and Godlewski (2013). Other recent papers using deep habits in lending Sharma (2023c,b,d,f,a,e).

Banking in DSGE models Marvin and McCallum (2007), collaterized borrowing with imperfectly competitive banking sector Iacoviello (2005), Kiyotaki and Moore (1997). Gerali, Neri, Sessa, and Signoretti (2010) banking sector with monopolistic competition of Dixit-Stiglitiz form. Andrés and Arce (2012) and Andrés, Arce, and Thomas (2013) incorporate Salop (1979) form of spatial competition.

3 Model

In model used in this paper features heterogeneous agents and credit limits. These features resemble setups used in Iacoviello (2005), Liu, Wang, and Zha (2013) and Justiniano, Primiceri, and Tambalotti (2015). In order to enable easy comparision with Ravn (2016), we closely follow his notation, wherever possible and highlight where the model in this paper departs from his. In contrast to his paper which paper which features households, entrepreneurs and banks, the model in this paper features two types of households – patient housheolds and impatient households. Both impatient households and entrepreneurs borrow in this model from banks whose only source of funding is deposits from patient households. The loan demand by households and entrepreneurs feature external habit à la Ravn, Schmitt-Grohé, and Uribe (2006). There are two types of households in the model who differ along their discount factor β. Patient households consume, supply labor and hold deposits with banks. They receive interest on their deposits and share of profits of banks. The impatient households also consume, supply labor and receive share of profits of the firms. But they borrow money from banks to hold housing which does not depreciate. Both types of households share the same preferences over consumption and housing. The consumption features external habits.

3.1 Patient Households

Households have the utility function of the following form:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^P\right)^t \left\{ \log\left(C_{i,t}^P - \gamma^P C_{i,t-1}^P\right) - \iota_t N_{i,t}^P + \varsigma_t \log H_{i,t}^P \right\} \tag{1}$$

where $C_{i,t}^P$, $N_{i,t}^P$ and $H_{i,t}^P$ denote consumption, labor and housing respectively of the patient households, $\beta^P \in (0,1)$ is a discount factor and γ^P measures the degree of habit formation in consumption. The superscript P denotes patient households. Households' preference for leisure is subject to an exogenous shock ι_t, the law of motion of which is given by

$$\log \iota_t = (1-\rho_N)\log \iota + \rho_N \log \iota_{t-1} + \sigma_N \epsilon_{N,t} \tag{2}$$

where $\epsilon_{N,t}$ is the iid innovation which follows a normal distribution with standard deviation σ_N and where $\iota > 0$ and $\rho_N \in (0,1)$. In similar fashion, ς_t is a housing preference shock as in Liu, Wang, and Zha (2013) which follows the following process

$$\log \varsigma_t = (1-\rho_H)\log \varsigma + \rho_H \log \varsigma_{t-1} + \sigma_H \epsilon_{H,t} \tag{3}$$

where $\sigma_{H,t}$ is the iid innovation which follows a normal distribution with standard deviation σ_H and where $\iota > 0$ and $\rho \in (0,1)$. The household faces the following budget constraint

$$C_{i,t}^P + Q_t^H\left(H_{i,t}^P - H_{i,t-1}^P\right) + \int_0^1 D_{ik,t}\mathrm{d}k \leq W_t^P N_{i,t}^P + \int_0^1 \Pi_{ik,t}\mathrm{d}k + R_{t-1}^D \int_0^1 D_{ik,t-1}\mathrm{d}k \tag{4}$$

Here, Q_t^H is the price of one unit of housing in terms of consumption goods, W_t^P is the real wage and R_{t-1}^D is the gross risk-free interest rate on the stock of deposits $D_{ik,t-1}$ of household i in bank k at the end of period $t-1$. I assume housing does not depreciate. Profits obtained by household i from bank k are denoted by $\Pi_{ik,t}$. After imposing symetric equilibrium, FOCs of the households can be written as

$$\frac{1}{C_t^P - \gamma^P C_{t-1}^P} - \beta^P \mathbb{E}_t \frac{\gamma^P}{C_{t+1}^P - \gamma^P C_t^P} = \lambda_t^P \tag{5}$$

$$\beta^P \mathbb{E}_t \lambda_{t+1}^P = \frac{\lambda_t^P}{R_t^D} \tag{6}$$

$$\frac{\varsigma_t}{H_t^P} + \beta^P \mathbb{E}_t \left(\lambda_{t+1}^P Q_{t+1}^H\right) = \lambda_t^P Q_t^H \tag{7}$$

$$\iota_t = \lambda_t^P W_t^P \tag{8}$$

First order conditions of the problem are derived in the Appendix A.

3.2 Impatient Households

I depart from Ravn (2016) by incorporating impatient households. This subsection describes their optimization problem. Impatient households have the utility function of the same form as the patient households. They, however, have a lower discount factor than patient households $\beta^I < \beta^P$.

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^I\right)^t \left\{ \log\left(C_{m,t}^I - \gamma^I C_{m,t-1}^I\right) - \iota_t N_{m,t}^I + \varsigma \log H_{m,t}^I \right\} \tag{9}$$

Unlike patient households, impatient households cannot buy houses outright, They instead rely on borrowing from banks to fund their buying of houses. Their borrowing is limited by a collateral constraint which is given by

$$l_{mk,t}^I \leq \frac{1}{R_{k,t}^L} \theta_{k,t} a_{m,t}^I \tag{10}$$

where $l_{mk,t}^I$ is the amount of loan the impatient househods can borrow, $R_{k,t}^L$ is the interest rate on that loan, $\theta_{k,t}$ is a loan-to-value (LTV) ratio required by the bank and $a_{m,t}^I$ is the asset owned by impateint households. The borrowing constraint makes it clear that borrowing by impatient households cannot exceed a fraction of the assets they hold. Impatient households hold all their assets in their housing and their total asset holding $a_{m,t}^I$ is given by

$$a_{m,t}^I = \mathbb{E}_t \left(Q_{t+1}^H H_{m,t}^I\right) \tag{11}$$

Impatient households borrow from banks to finance purchase of house and they display deep habits in banking relationships. Let $x_{m,t}^I$ denote impatient household m's 'habit-adjusted' borrowing. Since there is a continuum of bank in the economy who compete under monopolistic competition, this can be written as

$$x^I_{m,t} = \left[\int_0^1 \left(l^I_{mk,t} - \gamma^L s^I_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k\right]^{\frac{\xi}{\xi-1}} \tag{12}$$

where $\gamma^L \in (0,1)$ is the degree of habit formation in the demand for loans while $s^I_{k,t-1}$ is the stock of habits which evolves as

$$s^I_{k,t-1} = \rho_s s^I_{k,t-2} + (1-\rho_s)\, l^I_{k,t-1} \tag{13}$$

Given their total need for financing $x^I_{m,t}$, each household then chooses $l^I_{mk,t}$ so as to solve the following problem:

$$\min_{l^I_{mk,t}} \int_0^1 \Upsilon_{k,t} l^I_{mk,t} \mathrm{d}k \tag{14}$$

subject to collateral constraint (10) and their effective borrowing (12). Here, $\Upsilon_{k,t} \equiv R^L_{k,t} + \frac{\eta}{\theta_{k,t}}$ where the first term denotes the interest expenditure and the second term refers to value of pledged collateral. Impatient household m's optimal demand for loans from bank k is

$$l^I_{mk,t} = \left(\frac{\Upsilon_{k,t}}{\Upsilon_t}\right)^{-\xi} x^I_t + \gamma^L s^I_{k,t-1} \tag{15}$$

where $\Upsilon \equiv R^L_t + \eta \frac{1}{\theta_t}$ with $\theta_t = \left(\int_0^1 \theta_{k,t}^{1-\xi} \mathrm{d}k\right)^{\frac{1}{1-\xi}}$ representing the aggregate LTV ratio in the economy and $R^L_t \equiv \left[\int_0^1 (R^L_{k,t})^{1-\xi} \mathrm{d}k\right]^{\frac{1}{1-\xi}}$ the aggregate lending rate.

Impatient households maximize their utility by choosing their consumption, housing, labor and borrowing subject to their collateral constraint (10), asset holdings (11) and the following budget constraint:

$$C^I_{m,t} + \int_0^1 R^L_{k,t-1} l^I_{mk,t-1} \mathrm{d}k \leq W^I_t N^I_{m,t} - Q^H_t \left(H^I_{m,t} - H^I_{m,t-1}\right) + x^I_{m,t} + \Phi^I_t + \Psi^I_t \tag{16}$$

After imposing symmetric equilibrium, the FOCs of impatient households wrt consumption, housing, labor and loans respectively are:

$$\frac{1}{C_t^I - \gamma^I C_{t-1}^I} - \beta^I \mathbb{E}_t \frac{\gamma^I}{C_{t+1}^I - \gamma^I C_t^I} = \lambda_t^I \tag{17}$$

$$\frac{\varsigma_t}{H_t^I} + \mu_t^I \theta_t \mathbb{E}_t Q_{t+1}^H + \beta^I \mathbb{E}_t \left(\lambda_{t+1}^I Q_{t+1}^H\right) = \lambda_t^I Q_t^H \tag{18}$$

$$\iota_t = \lambda_t^I W_t^I \tag{19}$$

$$\beta^I \mathbb{E}_t \lambda_{t+1}^I R_t^L + \mu_t^I R_t^L = \lambda_t^I \tag{20}$$

All the derivations of first order conditions have been consigned to Appendix A.

3.3 Entrepreneurs

Entrepreneur j maximizes the utility obtained from consuming the non-durable consumption goods

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^E\right)^t \log\left(C_{j,t}^E - \gamma^E C_{j,t-1}^E\right) \tag{21}$$

where β^E and γ^E are as defined above. I assume that entrepreneurs are just as impatient as the impateint households, that is, $\beta^E = \beta^I < \beta^P$. Like impatient households, entrepreneurs also face a collateral constraint that limits the borrowing of each entrepreneur from each bank to a fraction of his assets

$$l_{jk,t}^E \leq \frac{1}{R_{k,t}^L} \theta_{k,t} a_{j,t}^E \tag{22}$$

Here, $l_{jk,t}^E$ denotes entrepreneur j's loan from bank k, expected value of entrepreneur's assets is $a_{j,t}^E$ and $R_{k,t}^L$ is the bank-specific lending rate. All entrepreneurs borrowing from bank k are subject to a loan-to-value (LTV) requirement $\theta_{k,t}$. In turn, $a_{j,t}^E$ is given by

$$a_{j,t}^E = \mathbb{E}_t \left(Q_{t+1}^H H_{j,t}^E + Q_{t+1}^K K_{j,t}\right) \tag{23}$$

In the above equation, Q_t^K denotes the value of installed capital in units of consumption goods, $K_{j,t}$ stock of capital and $H_{j,t}^E$ stock of housing.

Entrepreneurs have deep habits in banking relationships and and I let $x_{j,t}^E$ denote entrepreneur j's effective or habit-adjusted borrowing. Given the continuum of banks in the economy who compete under monopolistic competition, this can be written as

$$x_{j,t}^E = \left[\int_0^1 \left(l_{jk,t}^E - \gamma^L s_{k,t-1}^E \right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right]^{\frac{\xi}{\xi-1}} \tag{24}$$

where stock of habits $s_{k,t-1}$ evolves according to

$$s_{k,t-1}^E = \rho_s s_{k,t-2}^E + (1-\rho_s)\, l_{k,t-1}^E \tag{25}$$

Here, $\gamma^L \in (0,1)$ denotes the degree of habit formation in demand for loans and $\rho_s \in (0,1)$ measures the persistence of this habits. The parameter ξ denotes of the elasticity of substitution between loans from different banks and is thus a measure of the market power of each individual bank.

Given his total need for financing $x_{j,t}^E$, each entrepreneur chooses $l_{jk,t}^E$ to solve the following problem

$$\min_{l_{jk,t}^E} \int_0^1 \Upsilon_{k,t} l_{jk,t}^E \mathrm{d}k \tag{26}$$

subject to collateral constraint (22) and his effective borrowing (24). Here, $\Upsilon_{k,t} \equiv R_{k,t}^L + \frac{\eta}{\theta_{k,t}}$ where the first term denotes the interest expenditure and the second term refers to value of pledged collateral. Entrepreneur j's optimal demand for loans from bank k is

$$l_{jk,t}^E = \left(\frac{\Upsilon_{k,t}}{\Upsilon_t} \right)^{-\xi} x_t^E + \gamma^L s_{k,t-1}^E \tag{27}$$

where $\Upsilon \equiv R_t^L + \eta \frac{1}{\theta_t}$ with $\theta_t = \left(\int_0^1 \theta_{k,t}^{1-\xi} \mathrm{d}k \right)^{\frac{1}{1-\xi}}$ representing the aggregate LTV ratio in the economy and $R_t^L \equiv \left[\int_0^1 (R_{k,t}^L)^{1-\xi} \mathrm{d}k \right]^{\frac{1}{1-\xi}}$ the aggregate lending rate.

Production function of each entrepreneur is

$$Y_{j,t} = A_t \left\{ \left(N_{j,t}^P\right)^{\nu} \left(N_{m,t}^I\right)^{1-\nu} \right\}^{1-\alpha} \left\{ \left(H_{j,t-1}^E\right)^{\phi} \left(K_{j,t-1}\right)^{1-\phi} \right\}^{\alpha} \tag{28}$$

where $Y_{j,t}$ is output, $N_{i,t}^P$ and $N_{m,t}^I$ are labor inputs and $\alpha, \phi \in (0,1)$ are factor shares. TFP A_t follows the process

$$\log A_t = (1-\rho_A) \log A + \rho_A \log A_{t-1} + \sigma_A \epsilon_{A,t} \tag{29}$$

with iid innovation $\epsilon_{A,t}$ following a normal process with standard deviation σ_A where $A > 0$ and

$\rho_A \in (0,1)$. The evolution of capital obeys the following law of motion

$$K_{j,t} = (1-\delta) K_{j,t-1} + \left[1 - \frac{\Omega}{2}\left(\frac{I_{j,t}}{I_{j,t-1}} - 1\right)^2\right] I_{j,t} \tag{30}$$

where $I_{j,t}$ is firm j's investment level, $\delta \in (0,1)$ the rate of depreciation of capital stock and $\Omega > 0$ is a cost adjustment parameter. The entrepreneur faces the following budget constraint

$$C^E_{j,t} + \int_0^1 R^L_{k,t-1} l^E_{jk,t-1} \mathrm{d}k \leq Y_{j,t} - W^P_t N^P_{j,t} - W^I_t N^I_{j,t} - I_{j,t} - Q^H_t \left(H^E_{j,t} - H^E_{j,t-1}\right) + x^E_{j,t} + \Phi^E_t + \Psi^E_t \tag{31}$$

After imposing symmetric equilibrium, the FOCs of the entrepreneurs are

$$\lambda^E_t = \frac{1}{C^E_t - \gamma^E C^E_{t-1}} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C^E_{t+1} - \gamma^E C^E_t} \tag{32}$$

$$\lambda^E_t = \beta^E \mathbb{E}_t \lambda^E_{t+1} R^L_t + \mu^E_t R^L_t \tag{33}$$

$$W^P_t = (1-\alpha)\,\nu \frac{Y_t}{N^P_t} \tag{34}$$

$$W^I_t = (1-\alpha)(1-\nu) \frac{Y_t}{N^I_t} \tag{35}$$

$$\lambda^E_t Q^H_t = \beta^E \mathbb{E}_t \left[\lambda^E_{t+1}\left(Q^H_{t+1} + \alpha\phi \frac{Y_{t+1}}{H^E_t}\right)\right] + \mu^E_t \theta_t \mathbb{E}_t Q^H_{t+1} \tag{36}$$

$$\kappa^E_t = \alpha(1-\phi)\,\beta^E \mathbb{E}_t \left(\frac{\lambda^E_{t+1} Y_{t+1}}{K_t}\right) + \beta^E (1-\delta)\,\mathbb{E}_t \kappa^E_{t+1} + \mu^E_t \theta_t \mathbb{E}_t Q^K_{t+1} \tag{37}$$

$$\lambda^E_t = \kappa^E_t \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \Omega \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + \beta^E \Omega \mathbb{E}_t \left[\kappa^E_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 \left(\frac{I_{t+1}}{I_t} - 1\right)\right] \tag{38}$$

All derivations of first order conditions are contained in Appendix A.

3.4 Banking Sector

Banks in this model accept deposits from patient households and make loans to both impatient households and entrepreneurs. Banks take the interest rate on deposits R^D_t as given. Each individual bank k chooses its lending rate $R^L_{k,t}$, its LTV ratio $\theta_{k,t}$ and its total amount of lending $L_{k,t}$. The link between lower credit standards and higher credit risk is given as

$$p_{k,t} = \Xi + \varpi \left(\theta_{k,t} - \overline{\theta}\right) \tag{39}$$

Here, $p_{k,t}$ is bank-specific probability that a given loan is repaid and $\omega < 0$ measures the elasticity of this probability with respect to deviations of the bank's LTV ratio from its steady state level $\overline{\theta}$ which is same for all banks. Steady state repayment probability is given by $\Xi > 0$.

Each bank faces a standard trade-off when choosing its lending rate $R^L_{k,t}$. Profits of the bank k can be written as

$$\begin{aligned}
\Pi_{k,t} = & \left[\Xi + \varpi\left(\theta_{k,t-1} - \overline{\theta}\right)\right] R^L_{k,t-1} L^I_{k,t-1} + \left[\Xi + \varpi\left(\theta_{k,t-1} - \overline{\theta}\right)\right] R^L_{k,t-1} L^E_{k,t-1} \\
& + \left[1 - \Xi - \varpi\left(\theta_{k,t-1} - \overline{\theta}\right)\right] \frac{L^I_{k,t-1}}{\int_0^1 L^I_{k,t-1} \mathrm{d}k} \tau\theta_{t-1} a^I_{t-1} + \left[1 - \Xi - \varpi\left(\theta_{k,t-1} - \overline{\theta}\right)\right] \frac{L^E_{k,t-1}}{\int_0^1 L^E_{k,t-1} \mathrm{d}k} \tau\theta_{t-1} a^E_{t-1} \\
& + \int_0^1 D_{ik,t} \mathrm{d}i - L^I_{k,t} - L^E_{k,t} - R^D_{t-1} \int_0^1 D_{ik,t-1} \mathrm{d}i \\
= & \, p_t R^L_{k,t-1} L^I_{k,t-1} + p_t R^L_{k,t-1} L^E_{k,t-1} + (1 - p_t) \frac{L^I_{k,t-1}}{\int_0^1 L^I_{k,t-1} \mathrm{d}k} \tau\theta_{t-1} a^I_{t-1} + (1 - p_t) \frac{L^E_{k,t-1}}{\int_0^1 L^E_{k,t-1} \mathrm{d}k} \tau\theta_{t-1} a^E_{t-1} \\
& + \int_0^1 D_{ik,t} \mathrm{d}i - L^I_{k,t} - L^E_{k,t} - R^D_{t-1} \int_0^1 D_{ik,t-1} \mathrm{d}i
\end{aligned} \tag{40}$$

With probability $p_{k,t-1}$, the bank receives its loan back with interest. With complementary probability $(1 - p_{k,t-1})$, the loan is not reapid in which case bank k receives a share of the liquidation value of the borrower's total collaterized assets with its share given by its total lending relative to total lending of all other firms.

The balance sheet of bank k is simply

$$L_{k,t} = \int_0^1 D_{ik,t} \mathrm{d}i \tag{41}$$

where $L_{k,t}$ denotes total loans made by bank k to all impatient households and entrepreneurs, that is, $L_{k,t} \equiv \int_0^1 l^I_{mk,t} \mathrm{d}m + \int_0^1 l^E_{jk,t} \mathrm{d}j$. Each bank takes the demand for its loans as given

$$\begin{aligned}
L_{k,t} &= \int_0^1 l^I_{mk,t} \mathrm{d}m + \int_0^1 l^E_{jk,t} \mathrm{d}j \\
&= \int_0^1 \left[\left(\frac{\Upsilon_{k,t}}{\Upsilon_t}\right)^{-\xi} x^I_t + \gamma^L s^I_{k,t-1}\right] \mathrm{d}m + \int_0^1 \left[\left(\frac{\Upsilon_{k,t}}{\Upsilon_t}\right)^{-\xi} x^E_t + \gamma^L s^E_{k,t-1}\right] \mathrm{d}j
\end{aligned} \tag{42}$$

Each bank chooses $L^I_{k,t}$, $L^E_{k,t}$, $\theta_{k,t}$ and $R^L_{k,t}$ to maximize its profits subject to (41) and (42). Considering a symmetric equilibrium in which all banks optimally choose the same LTV ratio and the same lending rate, the FOCs for banks' optimization problem are:

$$\varrho_t^I = \mathbb{E}_t q_{t,t+1}\left[p_{k,t}R_{k,t}^L + (1-p_{k,t})\frac{\tau\theta_t a_t^I}{\int_0^1 L_{k,t}^I \mathrm{d}k} - R_t^D + \gamma^L(1-\rho_s)\mathbb{E}_t\varrho_{t+1}^I\right] \tag{43}$$

$$\varrho_t^E = \mathbb{E}_t q_{t,t+1}\left[p_{k,t}R_{k,t}^L + (1-p_{k,t})\frac{\tau\theta_t a_t^E}{\int_0^1 L_{k,t}^E \mathrm{d}k} - R_t^D + \gamma^L(1-\rho_s)\mathbb{E}_t\varrho_{t+1}^E\right] \tag{44}$$

$$\xi\varrho_t^I x_t^I \frac{\frac{\eta}{\theta_t}}{R_t^L\theta_t+\eta} + \xi\varrho_t^E x_t^E \frac{\frac{\eta}{\theta_t}}{R_t^L\theta_t+\eta} = -\varpi\mathbb{E}_t q_{t,t+1}\left(R_t^L L_t^I - \tau\theta_t a_t^I\right) - \varpi\mathbb{E}_t q_{t,t+1}\left(R_t^L L_t^E - \tau\theta_t a_t^E\right) \tag{45}$$

$$\xi\varrho_t^I x_t^I \frac{\theta_t}{\theta_t R_t^L+\eta} + \xi\varrho_t^E x_t^E \frac{\theta_t}{\theta_t R_t^L+\eta} = \mathbb{E}_t q_{t,t+1}p_{k,t}L_{k,t}^I + \mathbb{E}_t q_{t,t+1}p_{k,t}L_{k,t}^E \tag{46}$$

Derivation of all first order conditions have been relegated to Appendix A.

3.5 Aggregation and Market Clearing

Aggregate resource constraint of the economy is

$$C_t^P + C_t^I + C_t^E + I_t = Y_t \tag{47}$$

The clearing condition for the housing market is

$$H_t^P + H_t^I + H_t^E = H \tag{48}$$

where H is the total fixed supply of housing.

4 Equilibrium and Model Solution

The model is solved around its deterministic steady state using standard perturbation techniques (Adjemian, Bastani, Juillard, Karamé, Mihoubi, Mutschler, Pfeifer, Ratto, Rion, and Villemot, 2022). A period in the model is a quarter. Appendices B, C and D contain the list of equilibrium conditions, the list of steady-state conditions and the system of loglinear equations, respectively. The model is calibrated using parameter values standard in literature. The degree of habit formation is chosen to keep volatility of aggregate consumption relative to output consistent with US data. The resulting value of 0.6 is roughly close to estimates in the literature (Smets and Wouters, 2007). The patient households work about 15% of their time in steady state which dictates the choice of labor supply shock ι while the value of ς is chosen so as to obtain a ratio of residential land to output in steady state around 1.45 at annual frequency (Liu, Wang, and Zha, 2013).

Table 1: Parameter values

	Value	Description
β^s	0.995	Discount factor, patient households
β^i , $i = \{m, e\}$	0.95	Discount factor, impatient households and entrepreneurs
γ^i, $i = \{s, m, e\}$	0.6	Habits in consumption, patient/impatient households, entrepreneurs
ι	3.5	Steady state of labor supply shock
ς	0.0375	Steady state of housing preference shock
α	0.3	Non-labor share of production
ϕ	0.1	Land share of non-labor input
Ω	1.85	Investment adjustment cost parameter
δ	0.0285	Capital depreciation rate
τ	0.9432	Recovery rate of assets in liquidation
Ξ	0.98	Steady state of repayment probability
γ^L	0.72	Deep habit formation
ρ_s	0.93	Persistence of stock of deep habits
ξ	230	Elasticity of substitution between banks
ϖ	-1.5	Elasticity of credit risk
η	0.05	Weight of collateral minimization desire
ρ_A	0.95	Persistence of technology shock
ρ_N	0.97	Persistence of labor supply
ρ_H	0.99	Persistence of housing preference shock
σ_A	0.0014	Standard deviation of technology shock
σ_N	0.0014	Standard deviation of labor supply shock
σ_H	0.014	Standard deviation of housing preference shock

The labor income share takes a standard value of 0.3 which yields a steady-state capital-output ratio of 1.15, consistent with US data (Liu, Wang, and Zha, 2013). The input share of land in production is close to the value estimated by Liu, Wang, and Zha (2013) and in line with the value used in Iacoviello (2005). The available estimates for investment adjustment cost paramter range from close to 0 (Liu, Wang, and Zha, 2013) to above 26 (Christiano, Motto, and Rostagno, 2010). It is calibrated to allow volatility of non-residential investment relative to output to match its empirical counterpart which implies a value of 1.85. The rate of depreciation of capital is chosen to obtain a steady-state ratio of non-residential investment to output of slightly above 0.13 as consistent with US data (Beaudry and Lahiri, 2014). Following Liu, Wang, and Zha (2013), the recovery rate of assets in liquidation is calibrated to obtain an LTV ratio of 0.75 in steady state. The delinquency rate on commercial and industrial business loans in the US has fluctuated around an average close to 2% since mid 1990's. Using this, steady-state value of loan repayment probability Ξ is set to 0.98.

Following Aliaga-Díaz and Olivero (2010), the deep habit parameter in banking γ^L is set to 0.72. The persistence of stock of habits ρ_s is selected as to match the a duration of bank-firm relationship of 11 years as reported by Petersen and Rajan (1995). This is done by setting the persistence parameter ρ_s so that if the stock of habits $s_{k,t}$ were to increase exogeneously, only 5% of this increase would persist after 44 quarters. This implies a value of $\rho_s = 0.93$, rather close to the value of 0.85 used by Ravn, Schmitt-Grohé, and Uribe (2006) and Aliaga-Díaz and Olivero (2010). Elasticity of substitution between loans from different banks is calibrated so that interest rate spread between deposit and lending rates is 0.0168 in steady state (Aliaga-Díaz and Olivero, 2010). This implies an elasticity of substitution of 230 which is higher than elasticities of substituion usually employed in models of monopolistic competition in goods markets (Ravn, Schmitt-Grohé, and Uribe (2006) use a value of 5.3). Nevertheless, Aliaga-Díaz and Olivero (2010) argue that loans fro different banks are likely to be much better substitutes than products of different firms in the goods markets, as also reflected in much smaller observed markups. This suggests that elasticity of substitution should indeed be much higher. In fact, (aliaga2010macroeconomic) use an elasticity of substitution of 190 whereas Melina and Villa (2018) use a value of 427.

The parameter ϖ measures the elasticity of credit risk with respect to changes in LTV ratio. Using data from US mortgage loans originated between 1995 and 2008, excluding subprimes, Lam, Dunsky, and Kelly (2013) examine the impact of foreclosure and delinquency rates of higher LTV ratios at origination after controlling for borrower characteristics as well as housing and macroeconomic conditions. They report that foreclosure and delinquency rates tend to rise around one for one with the delinquency ratio, though this number differs between specifications. Von Furstenberg (1969) reports a higher elasticity 'in excess of unity'. The value of this elasticity is therefore chosen to be 1.5, that is $\varpi = -1.5$. Later, I conduct robustness checks regarding this parameter. Estimates of the value for η, entrepreneur's desire to minimize collateral pledges relative to cost minimization motive, are scarce. Booth and Booth (2006) find that firms' collateral minimization concern is of limited importance and they tend to choose the least costly form of borrowing. They point out that firms' willingness to accept higher lending rates in order ro reduce collateral requirements is rather small and therefore the value of η is set at 0.05 – a small value. The value of η turns out to be of limited quantitative importance as demonstrated later in the robustness analysis.

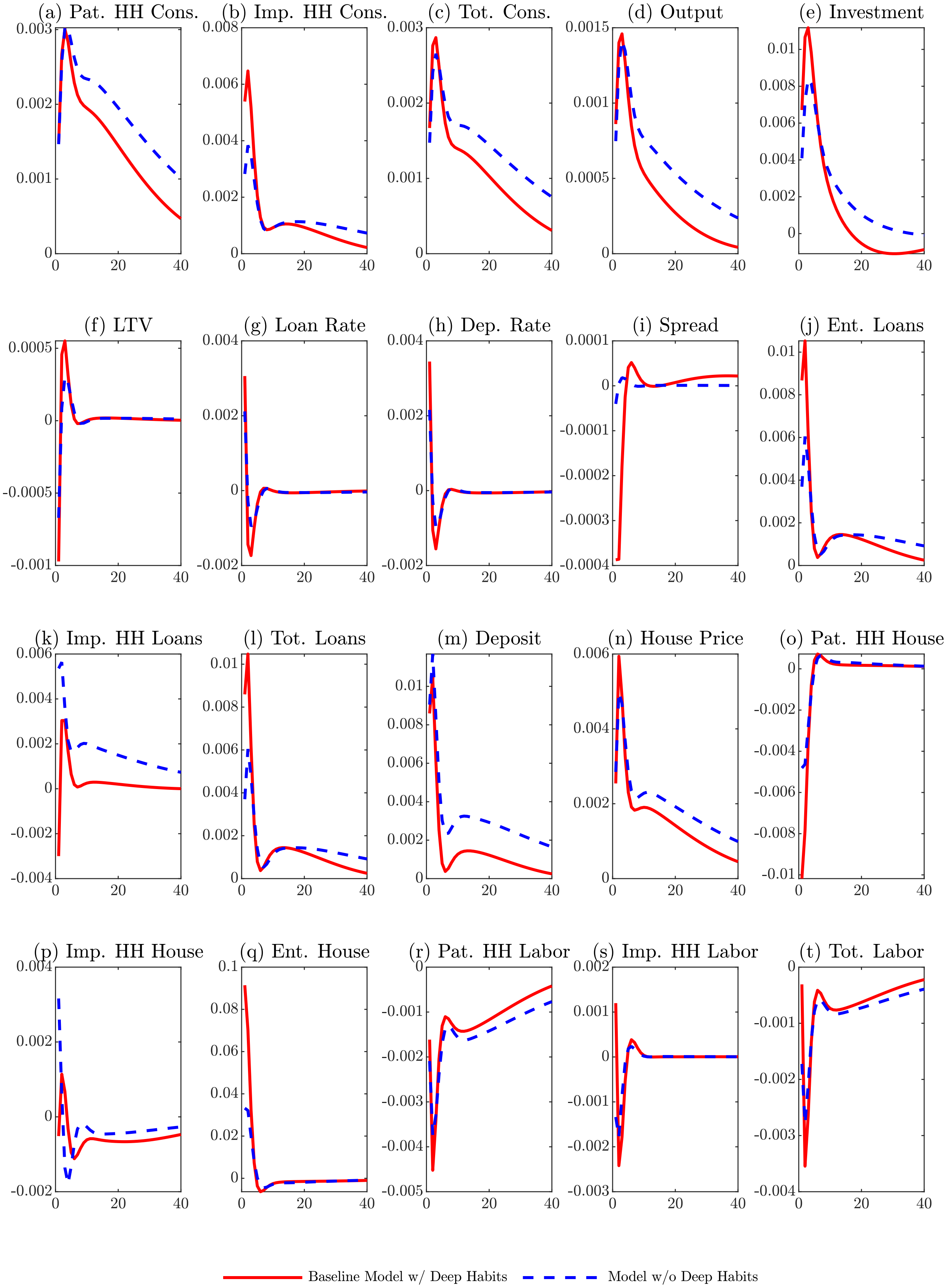
Following Smets and Wouters (2007), persistnce of technology shock σ_A is set to 0.95. Liu, Wang, and Zha (2013) include shocks to housing and labor supply in their study and find that housing preference shock is more persistent than technology shocks and the standard deviation of housing preference shock is roughly an order of magnitude larger than that of technology and labor supply shocks. The relative size of these shocks is therefore set accordingly.

5 Results

6 Conclusion

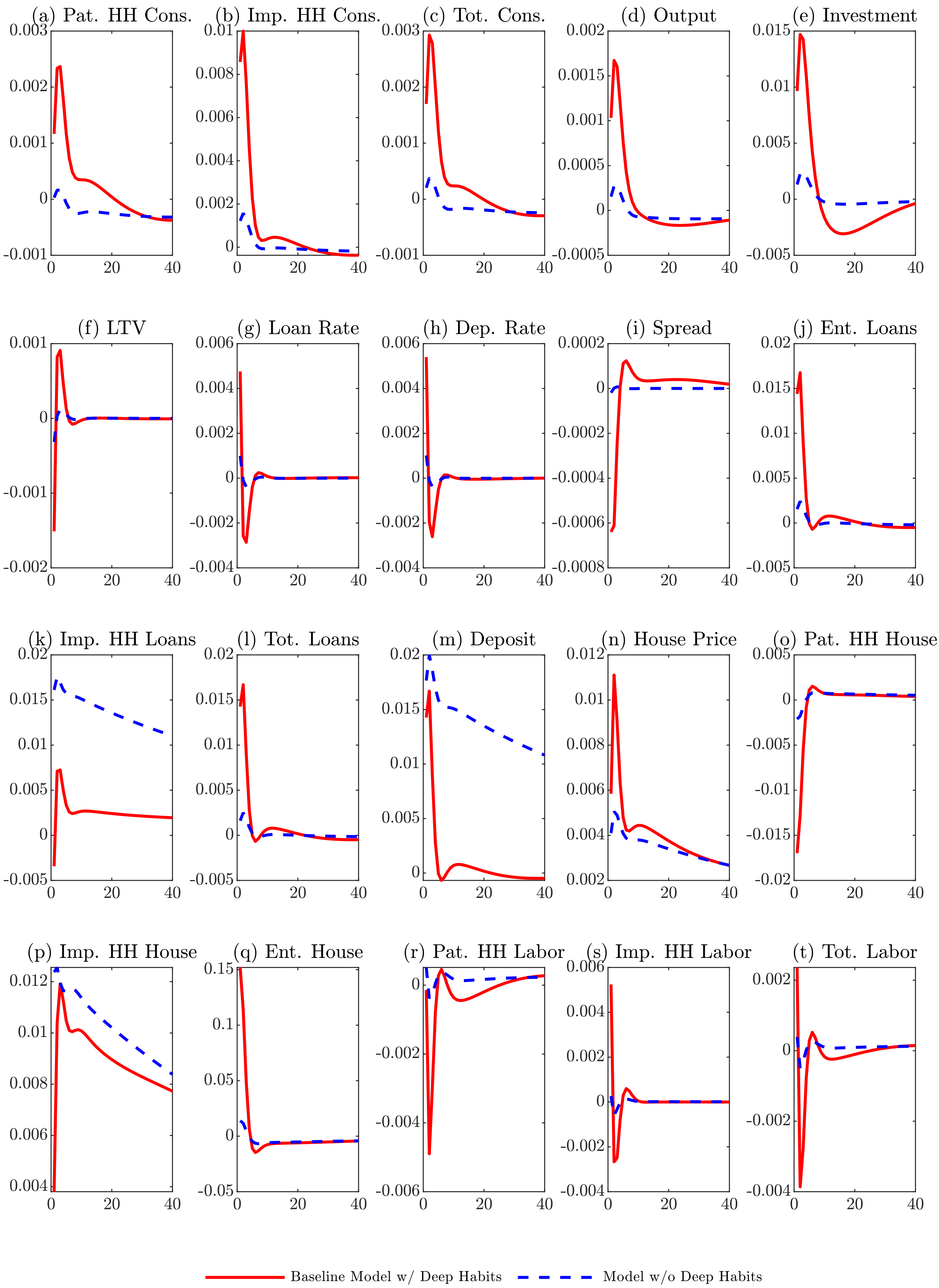
Household debt has been, for most part of last three decades, the largest component of overall debt in the US. This paper presented a model that shows that deep habits in household lending can explain sizable portion of business cycle variation in the US economy, above and beyond what is accounted for by business debt alone. Anylysis in this paper indicates that while deep habits in lending relationships explain sizable part of US economic fluctuations, leaving out household debt could result in underestimating the magnitude of impact deep habits in lending relationships can have on macroeconomic aggregates.

Figure 5: Impact of a positive TFP shock



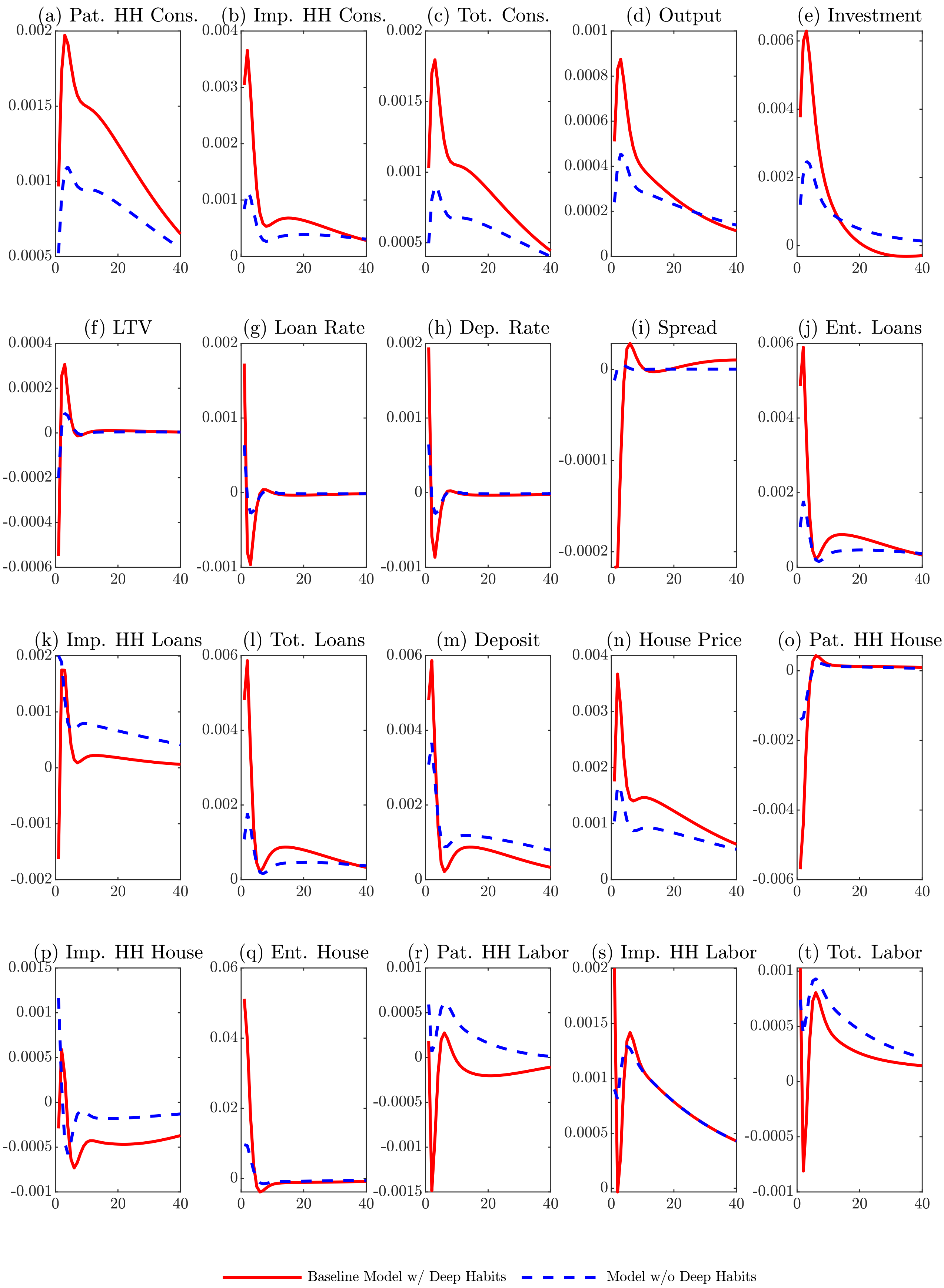
Note: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

Figure 6: Impact of a positive housing demand shock



Note: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

Figure 7: Impact of a positive labor shock



Note: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

References

Adjemian, S., H. Bastani, M. Juillard, F. Karamé, F. Mihoubi, W. Mutschler, J. Pfeifer, M. Ratto, N. Rion, and S. Villemot (2022): "Dynare: Reference Manual Version 5," Dynare Working Papers 72, CEPREMAP, https://ideas.repec.org/p/cpm/dynare/072.html.

Airaudo, M., M. P. Olivero, et al. (2014): "Optimal monetary policy with countercyclical credit spreads," *LeBow College of Business, Drexel University, Working Paper Series*, 1, 2014, https://archives.dynare.org/DynareConference2016/papers/OMP_DH_Sept_14_2016.pdf.

Aksoy, Y., H. S. Basso, and J. Coto-Martinez (2013): "Lending relationships and monetary policy," *Economic Inquiry*, 51, 368–393, https://doi.org/10.1111/j.1465-7295.2012.00453.x.

Aliaga-Díaz, R. and M. P. Olivero (2010): "Macroeconomic implications of "deep habits" in banking," *Journal of Money, Credit and Banking*, 42, 1495–1521, https://doi.org/10.1111/j.1538-4616.2010.00351.x.

Andrés, J. and O. Arce (2012): "Banking competition, housing prices and macroeconomic stability," *The Economic Journal*, 122, 1346–1372, https://doi.org/10.1111/j.1468-0297.2012.02531.x.

Andrés, J., O. Arce, and C. Thomas (2013): "Banking competition, collateral constraints, and optimal monetary policy," *Journal of Money, Credit and Banking*, 45, 87–125, https://doi.org/10.1111/jmcb.12072.

Avery, R. B., R. W. Bostic, and K. A. Samolyk (1998): "The role of personal wealth in small business finance," *Journal of Banking & Finance*, 22, 1019–1061, https://doi.org/10.1016/S0378-4266(98)00016-8.

Beaudry, P. and A. Lahiri (2014): "The Allocation of Aggregate Risk, Secondary Market Trades, and Financial Boom–Bust Cycles," *Journal of Money, Credit and Banking*, 46, 1–42, https://doi.org/10.1111/jmcb.12096.

Berger, A. N. and G. F. Udell (1995): "Relationship lending and lines of credit in small firm finance," *Journal of business*, 351–381, https://www.jstor.org/stable/2353332?seq=1.

Berlin, M. and A. W. Butler (2002): "Collateral and competition," Tech. rep., Federal Reserve Bank of Philadelphia, https://www.philadelphiafed.org/-/media/frbp/assets/working-papers/2002/wp02-22.pdf.

Bestor, H. (1985): "Screening versus rationing in credit markets with imperfect informatiom," *American Economic Review*, 75, 850–855, https://www.jstor.org/stable/1821362.

Booth, J. R. and L. C. Booth (2006): "Loan collateral decisions and corporate borrowing costs," *Journal of Money, Credit and Banking*, 67–90, https://www.jstor.org/stable/3839069.

Chakravarty, S., R. Feinberg, and E.-Y. Rhee (2004): "Relationships and individuals' bank switching behavior," *Journal of Economic Psychology*, 25, 507–527, https://doi.org/10.1016/S0167-4870(03)00051-5.

Christiano, L. J., R. Motto, and M. Rostagno (2010): "Financial factors in economic fluctuations," https://ssrn.com/abstract=1600166.

Dell'Ariccia, G. and R. Marquez (2006): "Lending booms and lending standards," *The Journal of Finance*, 61, 2511–2546, https://doi.org/10.1111/j.1540-6261.2006.01065.x.

Gerali, A., S. Neri, L. Sessa, and F. M. Signoretti (2010): "Credit and Banking in a DSGE Model of the Euro Area," *Journal of money, Credit and Banking*, 42, 107–141, https://doi.org/10.1111/j.1538-4616.2010.00331.x.

Gete, P. (2018): "Lending standards and macroeconomic dynamics," https://ssrn.com/abstract=3290435.

Hainz, C., L. Weill, and C. J. Godlewski (2013): "Bank competition and collateral: Theory and evidence," *Journal of Financial Services Research*, 44, 131–148, https://link.springer.com/content/pdf/10.1007/s10693-012-0141-3.pdf.

Iacoviello, M. (2005): "House prices, borrowing constraints, and monetary policy in the business cycle," *American economic review*, 95, 739–764, https://www.aeaweb.org/articles?id=10.1257/0002828054201477.

Jensen, H., S. H. Ravn, and E. Santoro (2018): "Changing credit limits, changing business cycles," *European Economic Review*, 102, 211–239, https://doi.org/10.1016/j.euroecorev.2017.12.008.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2015): “Household leveraging and deleveraging,” *Review of Economic Dynamics*, 18, 3–20, https://doi.org/10.1016/j.red.2014.10.003.

Kim, M., D. Kliger, and B. Vale (2003): “Estimating switching costs: the case of banking,” *Journal of Financial Intermediation*, 12, 25–56, https://doi.org/10.1016/S1042-9573(02)00005-0.

Kiyotaki, N. and J. Moore (1997): “Credit cycles,” *Journal of political economy*, 105, 211–248, https://doi.org/10.1086/262072.

Kocherlakota, N. R. (2000): “Creating business cycles through credit constraints,” *Federal Reserve Bank of Minneapolis Quarterly Review*, 24, 2–10, https://web.stanford.edu/~piazzesi/Reading/Kocherlakota%202000.pdf.

Lam, K., R. M. Dunsky, and A. Kelly (2013): “Impacts of down payment underwriting standards on loan performance—evidence from the GSEs and FHA portfolios,” *Federal Housing Finance Agency Working Paper*, 2013–12, https://www.fhfa.gov/PolicyProgramsResearch/Research/PaperDocuments/2013-12_WorkingPaper_13-3-508.pdf.

Liu, Z., P. Wang, and T. Zha (2013): “Land-price dynamics and macroeconomic fluctuations,” *Econometrica*, 81, 1147–1184, https://doi.org/10.3982/ECTA8994.

Marvin, G. and B. McCallum (2007): “Banking and Interest Rates in Monetary Policy Analysis,” *Journal of Monetary Economics*, https://doi.org/10.1016/j.jmoneco.2007.06.009.

Melina, G. and S. Villa (2014): “Fiscal policy and lending relationships,” *Economic Inquiry*, 52, 696–712, https://doi.org/10.1111/ecin.12051.

——— (2018): “Leaning against windy bank lending,” *Economic Inquiry*, 56, 460–482, https://doi.org/10.1111/ecin.12491.

Petersen, M. A. and R. G. Rajan (1994): “The benefits of lending relationships: Evidence from small business data,” *The journal of finance*, 49, 3–37, https://doi.org/10.1111/j.1540-6261.1994.tb04418.x.

——— (1995): “The effect of credit market competition on lending relationships,” *The Quarterly Journal of Economics*, 110, 407–443, https://doi.org/10.2307/2118445.

Rajan, R. G. (1994): "Why bank credit policies fluctuate: A theory and some evidence," *The Quarterly Journal of Economics*, 109, 399–441, https://doi.org/10.2307/2118468.

Ravn, M., S. Schmitt-Grohé, and M. Uribe (2006): "Deep habits," *The Review of Economic Studies*, 73, 195–218, https://doi.org/10.1111/j.1467-937X.2006.00374.x.

Ravn, S. H. (2016): "Endogenous credit standards and aggregate fluctuations," *Journal of Economic Dynamics and Control*, 69, 89–111, https://doi.org/10.1016/j.jedc.2016.05.011.

Ruckes, M. (2004): "Bank competition and credit standards," *Review of financial studies*, 17, 1073–1102, https://doi.org/10.1093/rfs/hhh011.

Salop, S. C. (1979): "Monopolistic competition with outside goods," *The Bell Journal of Economics*, 141–156, https://www.jstor.org/stable/3003323.

Sharma, V. (2023a): "Collateral shocks, lending relationships and economic dynamics," *CAMA Working Paper 49/2023*, https://cama.crawford.anu.edu.au/sites/default/files/publication/cama_crawford_anu_edu_au/2023-10/49_2023_sharma.pdf.

——— (2023b): "Credit shocks, endogenous lending standards and economic activity," *Unpublished Manuscript, University of Melbourne*, https://sharmavivek.com/assets/cselsmd.pdf.

——— (2023c): "Credit shocks, lending relationships and economic activity," *Unpublished Manuscript, University of Melbourne*, https://sharmavivek.com/assets/cslrea.pdf.

——— (2023d): "Financial shocks in a model of deep habits in banking," *Unpublished Manuscript, University of Melbourne*, https://sharmavivek.com/assets/fsdb.pdf.

——— (2023e): "Lending relationships and loan-to-value shocks," *Unpublished Manuscript, University of Melbourne*, https://sharmavivek.com/assets/lrltv.pdf.

——— (2023f): "Shocks to the lending standards and the macroeconomy," *CAMA Working Paper 55/2023*, https://cama.crawford.anu.edu.au/sites/default/files/publication/cama_crawford_anu_edu_au/2023-11/55_2023_sharma.pdf.

Sharpe, S. A. (1990): "Asymmetric information, bank lending, and implicit contracts: A stylized model of customer relationships," *The journal of finance*, 45, 1069–1087, https://doi.org/10.1111/j.1540-6261.1990.tb02427.x.

Smets, F. and R. Wouters (2007): "Shocks and frictions in US business cycles: A

Bayesian DSGE approach," *American economic review*, 97, 586–606, https://www.aeaweb.org/articles?id=10.1257/aer.97.3.586.

Stiglitz, J. E. and A. Weiss (1981): "Credit rationing in markets with imperfect information," *The American economic review*, 71, 393–410, https://www.jstor.org/stable/1802787.

Von Furstenberg, G. M. (1969): "Default risk on FHA-insured home mortgages as a function of the terms of financing: a quantitative analysis," *The Journal of Finance*, 24, 459–477, https://doi.org/10.1111/j.1540-6261.1969.tb00366.x.

Appendix (For Online Publication)

Twin Debts, Endogenous Lending Standards and Macroeconomic Fluctuations

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Contents

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A Derivation of FOCs

A.1 Patient Households

The Lagrangian of patient households is

$$\mathscr{L}_t = \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \left(\beta^P\right)^t \left[\begin{array}{c} \log\left(C_{i,t}^P - \gamma^P C_{i,t-1}^P\right) - \iota_t N_{i,t}^P + \varsigma_t \log H_{i,t}^P \\ -\lambda_{i,t}^P \left[\begin{array}{c} C_{i,t}^P + Q_t^H \left(H_{i,t}^P - H_{i,t-1}^P\right) + \int_0^1 D_{ik,t} \mathrm{d}k \\ -W_t^P N_{i,t}^P - \int_0^1 \Pi_{ik,t} \mathrm{d}k - R_{t-1}^D \int_0^1 D_{ik,t-1} \mathrm{d}k \end{array} \right] \end{array} \right] \right\} \tag{A.1}$$

The problem yields the following first order conditions (here, I ignore all the i's denoting individual patient households):

$$\frac{\partial \mathscr{L}}{\partial C_t^P} : \frac{1}{C_t^P - \gamma^P C_{t-1}^P} - \beta^P \mathbb{E}_P \frac{\gamma^P}{C_{t+1}^P - \gamma^P C_t^P} = \lambda_t^P \tag{A.2}$$

$$\frac{\partial \mathscr{L}}{\partial D_t} : \beta^P \mathbb{E}_t \lambda_{t+1}^P = \frac{\lambda_t^P}{R_t^D} \tag{A.3}$$

$$\frac{\partial \mathscr{L}}{\partial H_t^P} : \frac{\varsigma_t}{H_t^P} + \beta^P \mathbb{E}_t \left(\lambda_{t+1}^P Q_{t+1}^H\right) = \lambda_t^P Q_t^H \tag{A.4}$$

$$\frac{\partial \mathscr{L}}{\partial N_t^P} : \iota_t = \lambda_t^P W_t^P \tag{A.5}$$

A.2 Impatient Households

The impatient households solve their optimization probelm in two parts. The first part involves choosing how much to borrow from each individual bank $l_{mk,t}^I$ to minimize their total interest rate expenditure and the amount of collateral they have to post. This problem can be framed as

$$\min_{l_{mk,t}^I} \left[\int_0^1 R_{k,t}^L l_{mk,t}^I \mathrm{d}k + \eta \int_0^1 \frac{l_{mk,t}^I}{\theta_{k,t}} \mathrm{d}k \right] - \chi_t^I \left[x_{m,t}^I - \left(\int_0^1 \left(l_{mk,t}^I - \gamma^L s_{k,t-1}^I\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}k \right)^{\frac{\epsilon}{\epsilon-1}} \right] \tag{A.6}$$

This can be rewritten as

$$\min_{l_{mk,t}^I} \left[\int_0^1 \Upsilon_{k,t} l_{mk,t}^I \mathrm{d}k \right] - \chi_t^I \left[x_{m,t}^I - \left(\int_0^1 \left(l_{mk,t}^I - \gamma^L s_{k,t-1}^I\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}k \right)^{\frac{\epsilon}{\epsilon-1}} \right] \tag{A.7}$$

The first order condition for this problem is

$$R^L_{k,t} + \eta \frac{1}{\theta_{k,t}} = -\frac{\xi}{\xi-1}\chi^I_t \left(\int_0^1 \left(l^I_{mk,t} - \gamma^L s^I_{k,t-1} \right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{1}{\xi-1}} \frac{\xi-1}{\xi} \left(l^I_{mk,t} - \gamma^L s^I_{k,t-1} \right)^{-\frac{1}{\xi}} \tag{A.8}$$

The first order condition can be rewritten as

$$\begin{aligned}
R^L_{k,t} + \eta \frac{1}{\theta_{k,t}} &= -\chi^I_t \left(\int_0^1 \left(l^I_{mk,t} - \gamma^L s^I_{k,t-1} \right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{1}{\xi-1}} \left(l^I_{mk,t} - \gamma^L s^I_{k,t-1} \right)^{-\frac{1}{\xi}} \\
\left(R^L_{k,t} + \eta \frac{1}{\theta_{k,t}} \right) \left(l^I_{mk,t} - \gamma^L s^I_{k,t-1} \right) &= -\chi^I_t \left(\int_0^1 \left(l^I_{mk,t} - \gamma^L s^I_{k,t-1} \right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{1}{\xi-1}} \left(l^I_{mk,t} - \gamma^L s^I_{k,t-1} \right)^{\frac{\xi-1}{\xi}} \\
\int_0^1 \left(R^L_{k,t} + \eta \frac{1}{\theta_{k,t}} \right) \left(l^I_{mk,t} - \gamma^L s^I_{k,t-1} \right) \mathrm{d}k &= -\chi^I_t \left(\int_0^1 \left(l^I_{mk,t} - \gamma^L s^I_{k,t-1} \right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{1}{\xi-1}} \int_0^1 \left(l^I_{mk,t} - \gamma^L s^I_{k,t-1} \right)^{\frac{\xi-1}{\xi}} \mathrm{d}k
\end{aligned}$$

$$\int_0^1 R^L_{k,t} \left(l^I_{mk,t} - \gamma^L s^I_{k,t-1} \right) \mathrm{d}k + \eta \int_0^1 \frac{1}{\theta_{k,t}} \left(l^I_{mk,t} - \gamma^L s^I_{k,t-1} \right) \mathrm{d}k = -\chi^I_t \left(\int_0^1 \left(l^I_{mk,t} - \gamma^L s^I_{k,t-1} \right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{\xi}{\xi-1}}$$

$$x^I_{m,t} = -\frac{1}{\chi^I_t} \left[\int_0^1 R^L_{k,t} \left(l^I_{mk,t} - \gamma^L s^I_{k,t-1} \right) \mathrm{d}k + \eta \int_0^1 \frac{1}{\theta_{k,t}} \left(l^I_{mk,t} - \gamma^L s^I_{k,t-1} \right) \mathrm{d}k \right] \quad \ddagger$$

At the optimum, the following conditions must hold

$$\begin{aligned}
\frac{1}{\theta_t} x^I_{m,t} &= \int_0^1 \frac{1}{\theta_{k,t}} \left(l^I_{mk,t} - \gamma^L s^I_{k,t-1} \right) \mathrm{d}k \\
R^L_t x^I_{m,t} &= \int_0^1 R^L_{k,t} \left(l^I_{mk,t} - \gamma^L s^I_{k,t-1} \right) \mathrm{d}k
\end{aligned}$$

‡ can be rewritten as

$$\begin{aligned}
x^I_{m,t} &= -\frac{1}{\chi^I_t} \left[R^L_t x^I_{m,t} + \eta \frac{1}{\theta_t} x^I_{m,t} \right] \\
-\chi^I_t &= R^L_t + \eta \frac{1}{\theta_t}
\end{aligned}$$

Inserting this in first order condition

$$
\begin{aligned}
R^L_{k,t}+\eta\frac{1}{\theta_{k,t}} &= -\frac{\xi}{\xi-1}\chi^I_t\left(\int_0^1\left(l^I_{mk,t}-\gamma^L s^I_{k,t-1}\right)^{\frac{\xi-1}{\xi}}\mathrm{d}k\right)^{\frac{1}{\xi-1}}\frac{\xi-1}{\xi}\left(l^I_{mk,t}-\gamma^L s^I_{k,t-1}\right)^{-\frac{1}{\xi}}\\
R^L_{k,t}+\eta\frac{1}{\theta_{k,t}} &= \left(R^L_t+\eta\frac{1}{\theta_t}\right)\left(\int_0^1\left(l^I_{mk,t}-\gamma^L s^I_{k,t-1}\right)\mathrm{d}k\right)^{\frac{1}{\xi-1}}\left(l^I_{mk,t}-\gamma^L s^I_{k,t-1}\right)^{-\frac{1}{\xi}}\\
R^L_{k,t}+\eta\frac{1}{\theta_{k,t}} &= \left(R^L_{k,t}+\eta\frac{1}{\theta_t}\right)\left(x^I_t\right)^{\frac{1}{\xi}}\left(l^I_{mk,t}-\gamma^L s^I_{k,t-1}\right)^{-\frac{1}{\xi}}\\
\left(l^I_{mk,t}-\gamma^L s^I_{k,t-1}\right)^{\frac{1}{\xi}} &= \left(x^I_t\right)^{\frac{1}{\xi}}\frac{R^L_t+\eta\frac{1}{\theta_t}}{R^L_{k,t}+\eta\frac{1}{\theta_{k,t}}}\\
l^I_{mk,t} &= \left(\frac{R^L_t+\eta\frac{1}{\theta_t}}{R^L_{k,t}+\eta\frac{1}{\theta_{k,t}}}\right)^{\xi}x^I_t+\gamma^L s^I_{k,t-1}\\
l^I_{mk,t} &= \left(\frac{\Upsilon_t}{\Upsilon_{k,t}}\right)^{\xi}x^I_t+\gamma^L s^I_{k,t-1}\\
l^I_{mk,t} &= \left(\frac{\Upsilon_{k,t}}{\Upsilon_t}\right)^{-\xi}x^I_t+\gamma^L s^I_{k,t-1}
\end{aligned}
$$

When η is high, the impatient household attaches higher importance to collateral minimization motive. As a result, LTV ratios become more important for determination of demand for loans from each bank.

$$\lim_{\eta\to 0}\left(\frac{\Upsilon_{k,t}}{\Upsilon}\right)^{-\xi}=\lim_{\eta\to 0}\left(\frac{R^L_{k,t}+\eta\frac{1}{\theta_{k,t}}}{R^L_t+\eta\frac{1}{\theta_t}}\right)^{-\xi}=\left(\frac{R^L_{k,t}}{R^L_t}\right)^{-\xi}$$

The second part of impatient household's problem includes maximizing their utility, given their choice of loans. This can be written as

$$
\mathscr{L}_t=\mathbb{E}_t\left\{\sum_{t=0}^{\infty}\left(\beta^I\right)^t\left[\begin{array}{c}
\log\left(C^I_{m,t}-\gamma^I C^I_{m,t-1}\right)-\iota_t N^I_{m,t}+\varsigma_t\log H^I_{m,t}\\
-\lambda^I_{m,t}\left[\begin{array}{c}C^I_{m,t}+\int_0^1 R^L_{t-1}l^I_{mk,t-1}\mathrm{d}k-W^I_t N^I_{m,t}+Q^H_t\left(H^I_{m,t}-H^I_{m,t-1}\right)\\ -x^I_{m,t}-\Phi^I_t-\Psi^I_t\end{array}\right]\\
-\mu^I_{m,t}\left[R^L_{k,t}\int_0^1 l^I_{mk,t}\mathrm{d}k-\int_0^1\theta_{k,t}\mathrm{d}k\,\mathbb{E}_t\left(Q^H_{t+1}H^I_{m,t}\right)\right]\\
-\epsilon^I_{m,t}\left[x^I_{m,t}-\left\{\int_0^1\left(l^I_{mk,t}-\gamma^L s^I_{k,t-1}\right)^{\frac{\xi-1}{\xi}}\mathrm{d}k\right\}^{\frac{\xi}{\xi-1}}\right]
\end{array}\right]\right\} \tag{A.9}
$$

This program yields the following first order conditions (where I suppress all the m's denoting individual impatient households):

$$\frac{\partial \mathscr{L}}{\partial C_t^I} : \frac{1}{C_t^I - \gamma^I C_{t-1}^I} - \beta^I \mathbb{E}_t \frac{\gamma^I}{C_{t+1}^I - \gamma^I C_t^I} = \lambda_t^I \tag{A.10}$$

$$\frac{\partial \mathscr{L}}{\partial x_t^I} : \lambda_t^I = \epsilon_t^I \tag{A.11}$$

$$\frac{\partial \mathscr{L}}{\partial H_t^I} : \frac{\varsigma_t}{H_t^I} + \mu_t^I \theta_t \mathbb{E}_t Q_{t+1}^H + \beta^I \mathbb{E}_t \left(\lambda_{t+1}^I Q_{t+1}^H\right) = \lambda_t^I Q_t^H \tag{A.12}$$

$$\frac{\partial \mathscr{L}}{\partial N_t^I} : \iota_t = \lambda_t^I W_t^I \tag{A.13}$$

$$\frac{\partial \mathscr{L}}{\partial l_t^I} : \beta^I \mathbb{E}_t \lambda_{t+1}^I R_t^L + \mu_t^I R_t^L = \epsilon_t^I \tag{A.14}$$

Using $\lambda_t^I = \epsilon_t^I$ from (A.11), (A.14) can be written as

$$\beta^I \mathbb{E}_t \lambda_{t+1}^I R_t^L + \mu_t^I R_t^L = \lambda_t^I \tag{A.15}$$

A.3 Entrepreneurs

Entrepreneur's optimization problem features two parts, just like that of impatient households. The first part consists of choosing how much to borrow from each individual bank, $l_{jk,t}^E$ to minimize his total interest rate expenditure and amount of collateral he has to pledge. This part is identical to the first part of the optimization problem that Impatient husehold solve.

The second part of their optimization problem can be written as

$$\mathscr{L}_t = \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \left(\beta^E\right)^t \left[\begin{array}{c} \log\left(C_{j,t}^E - \gamma^E C_{j,t-1}^E\right) \\ -\lambda_{j,t}^E \left[\begin{array}{c} C_{j,t}^E + R_{k,t-1}^L \int_0^1 l_{jk,t-1}^E \mathrm{d}k - Y_{j,t} + W_t^E N_{j,t}^E + I_{j,t} \\ +Q_t^H \left(H_{j,t}^E - H_{j,t-1}^E\right) - x_{j,t}^E - \Phi_t^E - \Psi_t^E \end{array} \right] \\ -\mu_{j,t}^E \left[R_{k,t}^L \int_0^1 l_{jk,t}^E \mathrm{d}k - \int_0^1 \theta_{k,t} \mathrm{d}k \mathbb{E}_t \left(Q_{t+1}^H H_{j,t}^E + Q_{t+1}^K K_{j,t}\right)\right] \\ -\kappa_{j,t}^E \left[K_{j,t} - (1-\delta) K_{j,t-1} - \left\{1 - \frac{\Omega}{2}\left(\frac{I_{j,t}}{I_{j,t-1}} - 1\right)^2\right\} I_{j,t}\right] \\ -\epsilon_{j,t}^E \left[x_{j,t}^E - \left\{\int_0^1 \left(l_{jk,t}^E - \gamma^L s_{k,t-1}^E\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k\right\}^{\frac{\xi}{\xi-1}} \right] \end{array} \right] \right\} \tag{A.16}$$

where $Y_{j,t} = A_t \left\{ \left(N_{j,t}^P\right)^{\nu} \left(N_{j,t}^I\right)^{1-\nu} \right\}^{1-\alpha} \left\{ \left(H_{j,t-1}^E\right)^{\phi} \left(K_{j,t-1}\right)^{1-\phi} \right\}^{\alpha}$ may be inserted for $Y_{j,t}$ in the budget constraint. Solving entrepreneur's optmization problem, the first order conditions are (I ignore all j's here):

$$\frac{\partial \mathscr{L}}{\partial C_t^E}: \frac{1}{C_t^E - \gamma^E C_{t-1}^E} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C_{t+1}^E - \gamma^E C_t^E} = \lambda_t^E \tag{A.17}$$

$$\frac{\partial \mathscr{L}}{\partial x_t^E}: \lambda_t^E = \epsilon_t^E \tag{A.18}$$

$$\frac{\partial \mathscr{L}}{\partial l_t^E}: \epsilon_t^E = \beta^E \mathbb{E}_t \lambda_{t+1}^E R_t^L + \mu_t^E R_t^L \tag{A.19}$$

$$\frac{\partial \mathscr{L}}{\partial N_t^P}: W_t^P = \nu\left(1-\alpha\right)\frac{Y_t}{N_t^P} \tag{A.20}$$

$$\frac{\partial \mathscr{L}}{\partial N_t^I}: W_t^I = \left(1-\nu\right)\left(1-\alpha\right)\frac{Y_t}{N_t^I} \tag{A.21}$$

$$\frac{\partial \mathscr{L}}{\partial H_t^E}: \lambda_t^E Q_t^H = \beta^E \mathbb{E}_t\left\{\lambda_{t+1}^E\left(Q_{t+1}^H + \alpha\phi\frac{Y_{t+1}}{H_t^E}\right)\right\} + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^H \tag{A.22}$$

$$\frac{\partial \mathscr{L}}{\partial K_t}: \kappa_t^E = \alpha\left(1-\phi\right)\beta^E \mathbb{E}_t\left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t}\right) + \beta^E\left(1-\delta\right)\mathbb{E}_t \kappa_{t+1}^E + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^K \tag{A.23}$$

$$\frac{\partial \mathscr{L}}{\partial I_t}: \lambda_t^E = \kappa_t^E\left\{1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \Omega\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right\} + \beta^E \Omega \mathbb{E}_t\left\{\kappa_{t+1}^E\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - 1\right)\right\} \tag{A.24}$$

Using $\lambda_t^E = \epsilon_t^E$ from (A.18), (A.19) becomes

$$\beta^E \mathbb{E}_t\left(\lambda_{t+1}^E R_t^L\right) + \mu_t^E R_t^L = \lambda_t^E \tag{A.25}$$

A.4 Banks

The problem of banks is to choose their lending rate, LTV ratio and the total amount of lending. The bank considers deep habits in loan demand as well as adverse selection which is given by

$$p_{k,t} = \Xi + \varpi\left(\theta_{k,t} - \overline{\theta}\right) \tag{A.26}$$

The bank solves the following problem

$$\begin{aligned}
\max_{L_{k,t}^I, L_{k,t}^E, \theta_{k,t}, R_{k,t}^L} \Pi_t = &\left[\Xi + \varpi\left(\theta_{k,t} - \overline{\theta}\right)\right] R_{k,t-1}^L L_{k,t-1}^I + \left[1 - \Xi + \varpi\left(\theta_{k,t} - \overline{\theta}\right)\right]\frac{L_{k,t-1}^I}{\int_0^1 L_{k,t-1}^I \mathrm{d}k}\tau\theta_{t-1}a_{t-1}^I \\
&+ \left[\Xi + \varpi\left(\theta_{k,t} - \overline{\theta}\right)\right] R_{k,t-1}^L L_{k,t-1}^E + \left[1 - \Xi + \varpi\left(\theta_{k,t} - \overline{\theta}\right)\right]\frac{L_{k,t-1}^E}{\int_0^1 L_{k,t-1}^E \mathrm{d}k}\tau\theta_{t-1}a_{t-1}^E \\
&- R_{t-1}^D L_{k,t-1}^I + \varrho_t^I\left(\int_0^1\left[\left(\frac{R_t^L + \eta\frac{1}{\theta_t}}{R_{k,t}^L + \eta\frac{1}{\theta_{k,t}}}\right)^\xi x_t^I + \gamma^L s_{k,t-1}^I\right]\mathrm{d}m - L_{k,t}^I\right) \\
&- R_{t-1}^D L_{k,t-1}^E + \varrho_t^E\left(\int_0^1\left[\left(\frac{R_t^L + \eta\frac{1}{\theta_t}}{R_{k,t}^L + \eta\frac{1}{\theta_{k,t}}}\right)^\xi x_t^E + \gamma^L s_{k,t-1}^E\right]\mathrm{d}j - L_{k,t}^E\right)
\end{aligned}$$

The first order condition for $L_{k,t}^{I}$ is

$$\mathbb{E}_t q_{t,t+1} p_{k,t} R_{k,t}^{L} + \mathbb{E}_t q_{t,t+1} \left(1 - p_{k,t}\right) \frac{\tau \theta_t a_t^{I}}{\int_0^1 L_{k,t}^{I} \mathrm{d}k} - \mathbb{E}_t q_{t,t+1} R_t^{D} + \gamma^{L} \left(1 - \rho_s\right) \mathbb{E}_t \left(q_{t,t+1} \varrho_{t+1}^{I} - \varrho_t^{I}\right) = 0$$

$$\varrho_t^{I} = \mathbb{E}_t q_{t,t+1} \left[p_{k,t} R_{k,t}^{L} + \left(1 - p_{k,t}\right) \frac{\tau \theta_t a_t^{I}}{\int_0^1 L_{k,t}^{I} \mathrm{d}k} - R_t^{D} + \gamma^{L} \left(1 - \rho_s\right) \mathbb{E}_t \varrho_{t+1}^{I}\right] \tag{A.27}$$

The first order condition for $L_{k,t}^{E}$ is

$$\mathbb{E}_t q_{t,t+1} p_{k,t} R_{k,t}^{L} + \mathbb{E}_t q_{t,t+1} \left(1 - p_{k,t}\right) \frac{\tau \theta_t a_t^{E}}{\int_0^1 L_{k,t}^{E} \mathrm{d}k} - \mathbb{E}_t q_{t,t+1} R_t^{D} + \gamma^{L} \left(1 - \rho_s\right) \mathbb{E}_t \left(q_{t,t+1} \varrho_{t+1}^{E} - \varrho_t^{E}\right) = 0$$

$$\varrho_t^{E} = \mathbb{E}_t q_{t,t+1} \left[p_{k,t} R_{k,t}^{L} + \left(1 - p_{k,t}\right) \frac{\tau \theta_t a_t^{E}}{\int_0^1 L_{k,t}^{E} \mathrm{d}k} - R_t^{D} + \gamma^{L} \left(1 - \rho_s\right) \mathbb{E}_t \varrho_{t+1}^{E}\right] \tag{A.28}$$

The first order condition for $\theta_{k,t}$ is

$$\begin{aligned}
&\varpi \mathbb{E}_t q_{t,t+1} R_{k,t}^{L} L_{k,t}^{I} - \varpi \mathbb{E}_t q_{t,t+1} \frac{L_{k,t}^{I}}{\int_0^1 L_{k,t}^{I} \mathrm{d}k} \tau \theta_t a_t^{I} + \xi \varrho_t^{I} \left(\frac{R_t^{L} + \eta \frac{1}{\theta_t}}{R_{k,t}^{L} + \eta \frac{1}{\theta_{k,t}}}\right)^{\xi - 1} x_t^{I} \left(\frac{\eta \frac{1}{\theta_{k,t}^{2} \left(R_t^{L} + \eta \frac{1}{\theta_t}\right)}}{R_{k,t}^{L} + \eta \frac{1}{\theta_{k,t}}}\right)^{2} \\
&+ \varpi \mathbb{E}_t q_{t,t+1} R_{k,t}^{L} L_{k,t}^{E} - \varpi \mathbb{E}_t q_{t,t+1} \frac{L_{k,t}^{E}}{\int_0^1 L_{k,t}^{E} \mathrm{d}k} \tau \theta_t a_t^{E} + \xi \varrho_t^{E} \left(\frac{R_t^{L} + \eta \frac{1}{\theta_t}}{R_{k,t}^{L} + \eta \frac{1}{\theta_{k,t}}}\right)^{\xi - 1} x_t^{E} \left(\frac{\eta \frac{1}{\theta_{k,t}^{2} \left(R_t^{L} + \eta \frac{1}{\theta_t}\right)}}{R_{k,t}^{L} + \eta \frac{1}{\theta_{k,t}}}\right)^{2} = 0 \\
&\Rightarrow \xi \varrho_t^{I} x_t^{I} \left(\frac{R_t^{L} + \eta \frac{1}{\theta_t}}{R_{k,t}^{L} + \eta \frac{1}{\theta_{k,t}}}\right)^{\xi - 1} \frac{\eta \frac{1}{\theta_{k,t}^{2}} \left(R_t^{L} + \eta \frac{1}{\theta_t}\right)}{\left(R_{k,t}^{L} + \eta \frac{1}{\theta_{k,t}}\right)^{2}} + \xi \varrho_t^{E} x_t^{E} \left(\frac{R_t^{L} + \eta \frac{1}{\theta_t}}{R_{k,t}^{L} + \eta \frac{1}{\theta_{k,t}}}\right)^{\xi - 1} \frac{\eta \frac{1}{\theta_{k,t}^{2}} \left(R_t^{L} + \eta \frac{1}{\theta_t}\right)}{\left(R_{k,t}^{L} + \eta \frac{1}{\theta_{k,t}}\right)^{2}} \\
&= -\varpi \mathbb{E}_t q_{t,t+1} \left[R_{k,t}^{L} L_{k,t}^{I} - \frac{L_{k,t}^{I}}{\int_0^1 L_{k,t}^{I} \mathrm{d}k} \tau \theta_t a_t^{I}\right] - \varpi \mathbb{E}_t q_{t,t+1} \left[R_{k,t}^{L} L_{k,t}^{E} - \frac{L_{k,t}^{E}}{\int_0^1 L_{k,t}^{E} \mathrm{d}k} \tau \theta_t a_t^{E}\right]
\end{aligned} \tag{A.29}$$

The first order condition for $R_{k,t}^{L}$ is

$$\begin{aligned}
&\mathbb{E}_t q_{t,t+1} p_{k,t} L_{k,t}^{I} + \xi \varrho_t^{I} \left(\frac{R_t^{L} + \eta \frac{1}{\theta_t}}{R_{k,t}^{L} + \eta \frac{1}{\theta_t}}\right)^{\xi - 1} x_t^{I} \left(\frac{-\left(R_t^{L} + \eta \frac{1}{\theta_t}\right)}{\left(R_{k,t}^{L} + \eta \frac{1}{\theta_{k,t}}\right)^{2}}\right) \\
&\qquad + \mathbb{E}_t q_{t,t+1} p_{k,t} L_{k,t}^{E} + \xi \varrho_t^{E} \left(\frac{R_t^{L} + \eta \frac{1}{\theta_t}}{R_{k,t}^{L} + \eta \frac{1}{\theta_t}}\right)^{\xi - 1} x_t^{E} \left(\frac{-\left(R_t^{L} + \eta \frac{1}{\theta_t}\right)}{\left(R_{k,t}^{L} + \eta \frac{1}{\theta_{k,t}}\right)^{2}}\right) = 0
\end{aligned}$$

$$\Rightarrow \mathbb{E}_t q_{t,t+1} p_{k,t} L^I_{k,t} + \mathbb{E}_t q_{t,t+1} p_{k,t} L^E_{k,t} = \xi \varrho^I_t x^I_t \left(\frac{R^L_t + \eta \frac{1}{\theta_t}}{R^L_{k,t} + \eta \frac{1}{\theta_t}} \right)^{\xi - 1} \left(\frac{\left(R^L_t + \eta \frac{1}{\theta_t}\right)}{\left(R^L_{k,t} + \eta \frac{1}{\theta_{k,t}}\right)^2} \right)$$
$$+ \xi \varrho^E_t x^E_t \left(\frac{R^L_t + \eta \frac{1}{\theta_t}}{R^L_{k,t} + \eta \frac{1}{\theta_t}} \right)^{\xi - 1} \left(\frac{\left(R^L_t + \eta \frac{1}{\theta_t}\right)}{\left(R^L_{k,t} + \eta \frac{1}{\theta_{k,t}}\right)^2} \right) \tag{A.30}$$

In a symmetric equilibrium all banks have the same LTV ratio $\theta_{k,t} = \theta, \forall k$ and the same lending rate $R^L_{k,t} = R^L_t, \forall k$ and consequently lend the same amount $L^I_{k,t} = L^I_t, \forall k$ and $L^E_{k,t} = L^E_t, \forall k$. Bank's first order condition in this case can be rewritten as

$$\varrho^I_t = \mathbb{E}_t q_{t,t+1} \left[p_{k,t} R^L_{k,t} + (1 - p_{k,t}) \frac{\tau \theta_t a^I_t}{\int_0^1 L^I_{k,t} \mathrm{d}k} - R^D_t + \gamma^L (1 - \rho_s) \mathbb{E}_t \varrho^I_{t+1} \right] \tag{A.31}$$

$$\varrho^E_t = \mathbb{E}_t q_{t,t+1} \left[p_{k,t} R^L_{k,t} + (1 - p_{k,t}) \frac{\tau \theta_t a^E_t}{\int_0^1 L^E_{k,t} \mathrm{d}k} - R^D_t + \gamma^L (1 - \rho_s) \mathbb{E}_t \varrho^E_{t+1} \right] \tag{A.32}$$

$$\xi \varrho^I_t x^I_t \frac{\frac{\eta}{\theta}}{R^L_t \theta_t + \eta} + \xi \varrho^E_t x^E_t \frac{\frac{\eta}{\theta}}{R^L_t \theta_t + \eta} = -\varpi \mathbb{E}_t q_{t,t+1} \left(R^L_t L^I_t - \tau \theta_t a^I_t \right) - \varpi \mathbb{E}_t q_{t,t+1} \left(R^L_t L^E_t - \tau \theta_t a^E_t \right) \tag{A.33}$$

$$\xi \varrho^I_t x^I_t \frac{\theta_t}{\theta_t R^L_t + \eta} + \xi \varrho^E_t x^E_t \frac{\theta_t}{\theta_t R^L_t + \eta} = \mathbb{E}_t q_{t,t+1} p_{k,t} L^I_{k,t} + \mathbb{E}_t q_{t,t+1} p_{k,t} L^E_{k,t} \tag{A.34}$$

where I have imposed $L^I_t = l^I_t$ and $L^E_t = l^E_t$ in a symmetric equilibrium and that the collateral constraint is always binding (holds with equality at all times).

B List of Equations

B.1 Patient Households

$$\frac{1}{C^P_t - \gamma^P C^P_{t-1}} - \beta^P \mathbb{E}_t \frac{\gamma^P}{C^P_{t+1} - \gamma^P C^P_t} = \lambda^P_t \tag{B.1}$$

$$\beta^P \mathbb{E}_t \lambda^P_{t+1} = \frac{\lambda^P_t}{R^D_t} \tag{B.2}$$

$$\frac{\varsigma_t}{H^P_t} + \beta^P \mathbb{E}_t \left(\lambda^P_{t+1} Q^H_{t+1} \right) = \lambda^P_t Q^H_t \tag{B.3}$$

$$\iota_t = \lambda^P_t W^P_t \tag{B.4}$$

B.2 Impatient Households

$$\frac{1}{C^I_t - \gamma^I C^I_{t-1}} - \beta^I \mathbb{E}_t \frac{\gamma^I}{C^I_{t+1} - \gamma^I C^I_t} = \lambda^I_t \tag{B.5}$$

$$\frac{\varsigma_t}{H_t^I} + \mu_t^I \theta_t \mathbb{E}_t Q_{t+1}^H + \beta^I \mathbb{E}_t \left(\lambda_{t+1}^I Q_{t+1}^H \right) = \lambda_t^I Q_t^H \tag{B.6}$$

$$\iota_t = \lambda_t^I W_t^I \tag{B.7}$$

$$\beta^I \mathbb{E}_t \lambda_{t+1}^I R_t^L + \mu_t^I R_t^L = \lambda_t^I \tag{B.8}$$

$$s_t^I = \rho_s s_{t-1}^I + (1-\rho_s) l_t^I \tag{B.9}$$

$$x_t^I = \left(l_t^I - \gamma^L s_{t-1}^I \right) \tag{B.10}$$

$$L_t^I = l_t^I \tag{B.11}$$

$$C_t^I + R_{t-1}^L l_{t-1}^I = W_t^I N_t^I - Q_t \left(H_t^I - H_{t-1}^I \right) + x_t^I + \Phi_t^I + \Psi_t^I \tag{B.12}$$

$$l_t^I = \frac{\theta_t a_t^I}{R_t^L} \tag{B.13}$$

$$a_t^I = \mathbb{E}_t \left(Q_{t+1}^H H_t^I \right) \tag{B.14}$$

B.3 Entrepreneurs

$$\frac{1}{C_t^E - \gamma^E C_{t-1}^E} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C_{t+1}^E - \gamma^E C_t^E} = \lambda_t^E \tag{B.15}$$

$$\beta^E \mathbb{E}_t \left(\lambda_{t+1}^E R_t^L \right) + \mu_t^E R_t^L = \lambda_t^E \tag{B.16}$$

$$W_t^I = (1-\alpha)(1-\nu) \frac{Y_t}{N_t^I} \tag{B.17}$$

$$W_t^P = (1-\alpha) \nu \frac{Y_t}{N_t^P} \tag{B.18}$$

$$\lambda_t^E Q_t^H = \beta^E \mathbb{E}_t \left\{ \lambda_{t+1}^E \left(Q_{t+1}^H + \alpha \phi \frac{Y_{t+1}}{H_t^E} \right) \right\} + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^H \tag{B.19}$$

$$\kappa_t^E = \alpha (1-\phi) \beta^E \mathbb{E}_t \left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t} \right) + \beta^E (1-\delta) \mathbb{E}_t \kappa_{t+1}^E + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^K \tag{B.20}$$

$$\lambda_t^E = \kappa_t^E \left\{ 1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Omega \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) \right\} + \beta^E \Omega \mathbb{E}_t \left\{ \kappa_{t+1}^E \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \right\} \tag{B.21}$$

$$s_t^E = \rho_s s_{t-1}^E + (1-\rho_s) l_t^E \tag{B.22}$$

$$x_t^E = \left(l_t^E - \gamma^L s_{t-1}^E \right) \tag{B.23}$$

$$L_t^E = l_t^E \tag{B.24}$$

$$C_t^E + R_{t-1}^L l_{t-1}^E = Y_t - W_t^P N_t^P - W_t^I N_t^I - I_t - Q_t \left(H_t^E - H_{t-1}^E \right) + x_t^E + \Phi_t^E + \Psi_t^E \tag{B.25}$$

$$l_t^E = \frac{\theta_t a_t^E}{R_t^L} \tag{B.26}$$

$$a_t^E = \mathbb{E}_t \left(Q_{t+1}^H H_t^E + Q_{t+1}^K K_t \right) \tag{B.27}$$

$$\kappa_t^E = \lambda_t^E Q_t^K \tag{B.28}$$

B.4 Banks

$$\varrho_t^I = \mathbb{E}_t q_{t,t+1} \left[p_{k,t} R_{k,t}^L + (1 - p_{k,t}) \frac{\tau \theta_t a_t^I}{\int_0^1 L_{k,t}^I \mathrm{d}k} - R_t^D + \gamma^L (1 - \rho_s) \mathbb{E}_t \varrho_{t+1}^I \right] \tag{B.29}$$

$$\varrho_t^E = \mathbb{E}_t q_{t,t+1} \left[p_{k,t} R_{k,t}^L + (1 - p_{k,t}) \frac{\tau \theta_t a_t^E}{\int_0^1 L_{k,t}^E \mathrm{d}k} - R_t^D + \gamma^L (1 - \rho_s) \mathbb{E}_t \varrho_{t+1}^E \right] \tag{B.30}$$

$$\xi \varrho_t^I x_t^I \frac{\frac{\eta}{\theta}}{R_t^L \theta_t + \eta} + \xi \varrho_t^E x_t^E \frac{\frac{\eta}{\theta}}{R_t^L \theta_t + \eta} = -\varpi \mathbb{E}_t q_{t,t+1} \left(R_t^L L_t^I - \tau \theta_t a_t^I \right) - \varpi \mathbb{E}_t q_{t,t+1} \left(R_t^L L_t^E - \tau \theta_t a_t^E \right) \tag{B.31}$$

$$\xi \varrho_t^I x_t^I \frac{\theta_t}{\theta_t R_t^L + \eta} + \xi \varrho_t^E x_t^E \frac{\theta_t}{\theta_t R_t^L + \eta} = \mathbb{E}_t q_{t,t+1} p_{k,t} L_{k,t}^I + \mathbb{E}_t q_{t,t+1} p_{k,t} L_{k,t}^E \tag{B.32}$$

$$\begin{aligned} \Pi_{k,t} = {} & p_t R_{k,t-1}^L L_{k,t-1}^I + p_t R_{k,t-1}^L L_{k,t-1}^E + (1 - p_t) \frac{L_{k,t-1}^I}{\int_0^1 L_{k,t-1}^I \mathrm{d}k} \tau \theta_{t-1} a_{t-1}^I \\ & + (1 - p_t) \frac{L_{k,t-1}^E}{\int_0^1 L_{k,t-1}^E \mathrm{d}k} \tau \theta_{t-1} a_{t-1}^E + \int_0^1 D_{ik,t} \mathrm{d}i - L_{k,t}^I - L_{k,t}^E - R_{t-1}^D \int_0^1 D_{ik,t-1} \mathrm{d}i \end{aligned} \tag{B.33}$$

$$L_t^I + L_t^E = D_t \tag{B.34}$$

$$q_{t,t+1} = \beta^P \mathbb{E}_t \frac{\lambda_{t+1}^P}{\lambda_t^P} \tag{B.35}$$

$$p_t = \Xi + \varpi \left(\theta_t - \overline{\theta} \right) \tag{B.36}$$

B.5 Market Clearing and Resource Constraints

$$C_t^P + C_t^I + C_t^E + I_t = Y_t \tag{B.37}$$

$$H_t^P + H_t^I + H_t^E = H \tag{B.38}$$

$$Y_t = A_t \left\{ \left(N_t^P \right)^\nu \left(N_t^I \right)^{1-\nu} \right\}^{1-\alpha} \left\{ \left(H_{t-1}^E \right)^\phi \left(K_{t-1} \right)^{1-\phi} \right\}^\alpha \tag{B.39}$$

$$K_t = (1 - \delta) K_{t-1} + \left\{ 1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right\} I_t \tag{B.40}$$

C Steady State Conditions

All $i's$, $m's$, $j's$ and $k's$ denoting individual patient household, impatient household, entrepreneur and bank respectively are ignored. From patient household's FOC with respect to consumption (B.1) and labor (B.4), I have

$$\frac{1-\beta^P\gamma^P}{(1-\gamma^P)C^P} = \lambda^P, \tag{C.1}$$

and

$$\iota = \lambda^P W^P, \tag{C.2}$$

respectively. Patient household's FOC with respect to deposit (B.2) yields the steady-state gross interest rate

$$R^D = \frac{1}{\beta^P}, \tag{C.3}$$

underscoring that the time preference of most patient agent determines the steady-state rate of interest. From (B.3), I obtain

$$\begin{aligned} \frac{\varsigma}{H^P} + \beta^P\lambda^P Q^H &= \lambda^P Q^H, \\ \Rightarrow Q^H H^P &= \frac{\varsigma}{\lambda^P\big(1-\beta^P\big)}, \\ \Rightarrow H^P &= \frac{\varsigma}{Q^H\lambda^P\big(1-\beta^P\big)}. \end{aligned} \tag{C.4}$$

Turning to impatient households, from (B.5) and (B.7), I get

$$\frac{1-\beta^I\gamma^I}{(1-\gamma^I)C^I} = \lambda^I, \tag{C.5}$$

and

$$\iota = \lambda^I W^I, \tag{C.6}$$

respectively. Impatient household's FOC with respect to loans (B.8) yields

$$\begin{aligned} \beta^I\lambda^I R^L + \mu^I R^L &= \lambda^I, \\ \Rightarrow \mu^I &= \frac{\lambda^I\big(1-\beta^I R^L\big)}{R^L}. \end{aligned} \tag{C.7}$$

This equation shows that collateral contraint of the impatient household is binding in the steady state, that is, μ^I is positive if and only if the discount factor of impatient households β^I is less than $1/R^L$. In the baseline calibration, I set β^I to 0.95 while steady state value of R^L is 1.0219 which indicates that β^I must be lower than 0.9786 which is actually the case since the value of β^I used here is 0.95. Impatient household's FOC with respect to housing (B.6) gives

$$\begin{aligned} \frac{\varsigma}{H^I} + \beta^I \lambda^I Q^H &= \lambda^I Q^H + \mu^I \theta Q^H, \\ \Rightarrow H^I &= \frac{\varsigma}{\lambda^I Q^H (1-\beta^I) + \mu^I \theta Q^H}, \\ &= \frac{\varsigma}{\lambda^I Q^H \left[(1-\beta^I) + \frac{(1-\beta^I R^L)}{R^L} \theta \right]}. \end{aligned} \tag{C.8}$$

I next turn to entrepreneurs. Their consumption FOC (B.15) yields

$$\frac{1-\beta^E \gamma^E}{(1-\gamma^E) C^E} = \lambda^E \tag{C.9}$$

Entrepreneur's FOC with respect to loans (B.16) gives

$$\begin{aligned} \beta^E \lambda^E R^L + \mu^E R^L &= \lambda^E \\ \Rightarrow \mu^E &= \frac{\lambda^E (1-\beta^E R^L)}{R^L} \end{aligned} \tag{C.10}$$

Just as I have shown for impatient households, the borrowing constraint for entrepreneurs binds if and only if μ^E is positive. This implies that β^E must be less than R^L. In the baseline calibration, β^E is set to 0.95 whereas the steady state value of R^L is 1.0219 which implies that β^E must be less than 0.9786 which is indeed the case. Entrepreneur's production function is

$$Y = A \left[(N^P)^{\nu} (N^I)^{1-\nu} \right]^{1-\alpha} \left[(H^E)^{\phi} (K)^{1-\phi} \right]^{\alpha} \tag{C.11}$$

Firm's labor choice for patient househods (B.18) yields

$$W^P = \nu (1-\alpha) \frac{Y}{N^P} \tag{C.12}$$

From firm's labor choice for impatient households (B.17), I have

$$W^I = (1-\nu)(1-\alpha) \frac{Y}{N^I} \tag{C.13}$$

From entrepreneur's FOC with respect to housing (B.19), I obtain

$$\lambda^E Q^H = \beta^E \lambda^E \left(Q^H + \alpha\phi \frac{Y}{H^E} \right) + \mu^E \theta Q^H$$
$$\Rightarrow \frac{Q^H H^E}{Y} = \frac{\beta^E \alpha\phi R^L}{\left(1-\beta^E\right) R^L - \theta\left(1-\beta^E R^L\right)} \tag{C.14}$$

Aggregate law of motion for capital (B.40) gives

$$K = (1-\delta)K + \left[1 - \frac{\Omega}{2}\left(\frac{I}{I} - 1\right)\right] I$$
$$\Rightarrow I = \delta K \tag{C.15}$$

I have the following steady-state resource constraints

$$Y = C^P + C^I + C^E + I \tag{C.16}$$

$$H = H^P + H^I + H^E \tag{C.17}$$

$$D = L^I + L^E \tag{C.18}$$

Also, I have the following steady-state version of agents' budget constraints (one of them is redundant because of Walras' Law)

$$C^P = W^P N^P - (R^D - 1)D + \Pi \tag{C.19}$$

$$C^I = W^I N^I - R^L l^I + x^I + \Phi^I + \Psi^I \tag{C.20}$$

$$C^E = Y - R^L l^E - W^P N^P - W^I N^I - I - x^E - \Phi^E - \Psi^E \tag{C.21}$$

So the steady state is characterized by the vector

$$\left[Y, C^P, C^I, C^E, I, H^P, H^I, H^E, K, N^P, N^I, L^I, L^E, D, Q^H, Q^K, R^D, R^L, W^P, W^I, \lambda^P, \lambda^I, \lambda^E, \mu^I, \mu^E\right]$$

From entrepreneur's optimal choice of capital (B.20), I have

$$\kappa_t^E = \alpha\left(1-\alpha\right)\beta^E \mathbb{E}_t\left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t}\right) + \beta^E\left(1-\delta\right)\mathbb{E}_t \kappa_{t+1}^E + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^K$$
$$\Rightarrow \frac{\kappa^E}{\lambda^E}\left(1 - \left(1-\delta\right)\beta^E\right) = \alpha\left(1-\phi\right)\beta^E \frac{Y}{K} + \frac{\left(1-\beta^E R^L\right)}{R^L}\theta Q^K \tag{C.22}$$

Entrepreneur's optimal choice of investment (B.21) yields

$$
\begin{aligned}
\lambda_t^E(j) = \kappa_t^E(j) & \left[1 - \frac{\Omega}{2}\left(\frac{I_t(j)}{I_t(j-1)} - 1\right)^2 - \Omega \frac{I_t(j)}{I_t(j-1)}\left(\frac{I_t(j)}{I_{t-1}(j)} - 1\right)\right] \\
& + \beta^E \Omega \mathbb{E}_t\left[\kappa_{t+1}^E(j)\left(\frac{I_{t+1}(j)}{I_t(j)}\right)^2\left(\frac{I_{t+1}(j)}{I_t(j)} - 1\right)\right] \\
\Rightarrow \lambda^E = \kappa^E &
\end{aligned}
\tag{C.23}
$$

Combining this with steady state version of

$$
\kappa^E = \lambda^E Q^K \tag{C.24}
$$

I obtain $Q^K = 1$ in the steady state. Plugging this into (C.22), I obtain the expression for capital-to-output ratio

$$
\begin{aligned}
\frac{\kappa^E}{\lambda^E}\left(1 - (1-\delta)\beta^E\right) &= \alpha(1-\phi)\beta^E \frac{Y}{K} + \frac{\left(1 - \beta^E R^L\right)}{R^L}\theta Q^K \\
\Rightarrow \frac{K}{Y} &= \frac{\alpha(1-\phi) R^L \beta^E}{R^L\left(1 - (1-\delta)\beta^E\right) - \theta\left(1 - \beta^E R^L\right)}
\end{aligned}
\tag{C.25}
$$

Next, combining (B.26) and (B.27)

$$
l^E = \frac{\theta}{R^L}\left[Q^H H^E + Q^K K\right] \tag{C.26}
$$

Dividing by Y, the above expression becomes

$$
\frac{l^E}{Y} = \frac{\theta}{R^L}\left[\frac{Q^H H^E}{Y} + \frac{Q^K K}{Y}\right]
$$

Plugging in the values of $\frac{Q^H H^E}{Y}$ and $\frac{Q^K K}{Y}$, I obtain entrepreneur's debt-to-output ratio

$$
\frac{l^E}{Y} = \alpha\theta\beta^E\left[\frac{\phi}{R^L\left(1 - \beta^E\right) - \theta\left(1 - \beta^E R^L\right)} + \frac{(1-\phi)}{R^L\left(1 - (1-\delta)\beta^E\right) - \theta\left(1 - \beta^E R^L\right)}\right] \tag{C.27}
$$

From entrepreneur's budget constraint (B.25)

$$
C^E + R^L l^E = Y - W^P N^P - W^I N^I - I + x^E + \Phi^E + \Psi^E \tag{C.28}
$$

Rewriting this in ratios to output

$$
\begin{aligned}
\frac{C^E}{Y}+\frac{R^L l^E}{Y}&=1-\frac{W^P N^P}{Y}-\frac{W^I N^I}{Y}-\frac{I}{Y}+\frac{x^E}{Y}+\frac{\Phi^E}{Y}+\frac{\Psi^E}{Y}\\
\Rightarrow\frac{C^E}{Y}&=\alpha-\delta\frac{K}{Y}+\left(1-\gamma^L-R^L\right)\frac{l^E}{Y}+\frac{\Phi^E}{Y}+\frac{\Psi^E}{Y}
\end{aligned}
\tag{C.29}
$$

where steady-state expressions for W^P, W^I and x^E have been used. Now, using the steady-state expressions for Φ^E and Ψ^E

$$
\begin{aligned}
\frac{C^E}{Y}&=\alpha-\delta\frac{K}{Y}+\left(1-\gamma^L-R^L\right)\frac{l^E}{Y}+\frac{\gamma^L s^E}{Y}+\frac{(1-p)\left(R^L L^E-\tau\theta a^E\right)}{Y}\\
\Rightarrow\frac{C^E}{Y}&=\alpha-\delta\frac{K}{Y}+\left[1-pR^L-(1-p)\,\tau R^L\right]\frac{l^E}{Y}
\end{aligned}
\tag{C.30}
$$

Using the budget constraint of impatient households (B.12)

$$
C^I+R^L l^I=W^I N^I+x^I+\Phi^I+\Psi^I \tag{C.31}
$$

Dividing by Y,

$$
\frac{C^I}{Y}+\frac{R^L l^I}{Y}=\frac{W^I N^I}{Y}+\frac{x^I}{Y}+\frac{\Phi^I}{Y}+\frac{\Psi^I}{Y} \tag{C.32}
$$

Using the steady state expressions for Φ^I and Ψ^I,

$$
\frac{C^I}{Y}=(1-\nu)(1-\alpha)+\left(1-\gamma^L-R^L\right)\frac{l^I}{Y}+\frac{\gamma^L s^I}{Y}+\frac{(1-p)\left(R^L L^I-\tau\theta a^I\right)}{Y} \tag{C.33}
$$

Upon simplification,

$$
\frac{C^I}{Y}=(1-\nu)(1-\alpha)+\left(1-pR^L-(1-p)\,\tau R^L\right)\frac{l^I}{Y} \tag{C.34}
$$

Writing impatient household's debt limit (B.13) in the form of ratio,

$$
\frac{l^I}{Y}=\frac{\theta}{R^L}\left(\frac{Q^H H^I}{Y}\right) \tag{C.35}
$$

Steady-state budget constraint of patient household, in ratio to output, reads

$$
\begin{aligned}
\frac{C^P}{Y}&=\frac{W^P N^P}{Y}+\left(R^D-1\right)\frac{D}{Y}+\frac{\Pi}{Y}\\
&=\nu(1-\alpha)+\frac{\left(R^D-1\right)\left(L^E+L^I\right)}{Y}+\frac{\left(pR^L-R^D\right)\left(L^E+L^I\right)+(1-p)\,\tau\theta\left(a^E+a^I\right)}{Y}
\end{aligned}
\tag{C.36}
$$

Adding the budget constraints of patient and impatient households

$$\frac{C^P + C^I}{Y} = (1-\alpha) + (1-p)\,\tau\theta\left(a^I + a^E\right) - \left[(1-p)\,\tau R^L\right]\frac{l^I}{Y} - \left(1 - pR^L\right)\frac{l^E}{Y} \tag{C.37}$$

Using borrowing constraint of impatient households and entrepreneurs, that is, $l^I = \frac{\theta}{R^L}a^I \Rightarrow a^I = \frac{l^I R^L}{\theta}$ and $l^E = \frac{\theta}{R^L}a^E \Rightarrow a^E = \frac{l^E R^L}{\theta}$ respectively, the above equation can be rewritten as

$$\frac{C^P + C^I}{Y} = (1-\alpha) + \left[(1-p)\,\tau R^L - \left(1 - pR^L\right)\right]\frac{l^E}{Y} \tag{C.38}$$

$$\frac{C^I}{Y} = (1-\nu)(1-\alpha) + \left[1 - pR^L - (1-p)\,\tau R^L\right]\frac{\theta}{R^L}\left(\frac{Q^H H^I}{Y}\right) \tag{C.39}$$

Dividing the above two expressions by each other, I have

$$\begin{aligned}
\frac{\frac{Q^H H^P}{Y}}{\frac{Q^H H^E}{Y}} &= \frac{\frac{\varsigma}{Y\lambda^P\left(1-\beta^P\right)}}{\frac{\beta^E\alpha\phi R^L}{\left(1-\beta^P\right)R^L - \theta\left(1-\beta^E R^L\right)}} \\
\Rightarrow \frac{H^P}{H^E} &= \frac{\varsigma}{Y\frac{1-\beta^P\gamma^P}{\left(1-\gamma^P\right)C^P}\left(1-\beta^P\right)}\frac{\left(1-\beta^E\right)R^L - \theta\left(1-\beta^E R^L\right)}{\beta^E\alpha\phi R^L} \\
\Rightarrow \frac{H^P}{H - H^P} &= \frac{\varsigma\left(1-\gamma^P\right)}{\left(1-\beta^P\right)\left(1-\beta^P\gamma^P\right)}\frac{\left(1-\beta^P\right)R^L - \theta\left(1-\beta^E R^L\right)}{\beta^E\alpha\phi R^L}\frac{C^P}{Y}
\end{aligned} \tag{C.40}$$

Impatient household's stock of habits for loans (B.9) implies

$$\begin{aligned}
s_t^I &= \rho_s s_{t-1}^I + (1-\rho_s)\, l_t^I \\
s^I &= l^I
\end{aligned} \tag{C.41}$$

In similar fashion, entrepreneur's stock of habits for loans (B.22) leads to

$$\begin{aligned}
s_t^E &= \rho_s s_{t-1}^E + (1-\rho_s)\, l_t^E \\
s^E &= l^E
\end{aligned} \tag{C.42}$$

Impatient household's effective demand (habit-adjusted) for loans (B.10) yields

$$\begin{aligned}
x_t^I &= \left(l_t^I - \gamma^L s_{t-1}^I\right) \\
\Rightarrow x^I &= \left(1-\gamma^L\right) l^I
\end{aligned} \tag{C.43}$$

Likewise, entrepreneur's effective demand for loans (B.23) writes

$$x_t^E = \left(l_t^E - \gamma^L s_{t-1}^E\right)$$
$$\Rightarrow x^E = \left(1 - \gamma^L\right) l^E \tag{C.44}$$

Total loans of impatient households (B.11) equal loans given to all impatient households by all banks

$$L^I = l^I \tag{C.45}$$

Similarly, total loans of entrepreneurs (B.24) are equal to loans given to all entrepreneurs by all banks

$$L^E = l^E \tag{C.46}$$

From bank's balance sheet condition (B.34), deposits must equal total loans to impatient households and entrepreneurs

$$D = L^I + L^E \tag{C.47}$$

Steady-state version of bank's stochastic discount factor (B.35) reads

$$q = \beta^P \tag{C.48}$$

I collect here bank's first order conditions (B.29), (B.30), (B.31) and (B.32)

$$\varrho_t^I = \mathbb{E}_t q_{t,t+1}\left[p_{k,t} R_{k,t}^L + (1 - p_{k,t}) \frac{\tau\theta_t a_t^I}{\int_0^1 L_{k,t}^I \mathrm{d}k} - R_t^D + \gamma^L (1 - \rho_s) \mathbb{E}_t \varrho_{t+1}^I\right] \tag{C.49}$$

$$\varrho_t^E = \mathbb{E}_t q_{t,t+1}\left[p_{k,t} R_{k,t}^L + (1 - p_{k,t}) \frac{\tau\theta_t a_t^E}{\int_0^1 L_{k,t}^E \mathrm{d}k} - R_t^D + \gamma^L (1 - \rho_s) \mathbb{E}_t \varrho_{t+1}^E\right] \tag{C.50}$$

$$\xi \varrho_t^I x_t^I \frac{\frac{\eta}{\theta}}{R_t^L \theta + \eta} + \xi \varrho_t^E x_t^E \frac{\frac{\eta}{\theta}}{R_t^L \theta + \eta} = -\varpi \mathbb{E}_t q_{t,t+1}\left[R_t^L L_t^I - \tau\theta_t a_t^I\right] - \varpi \mathbb{E}_t q_{t,t+1}\left[R_t^L L_t^E - \tau\theta_t a_t^E\right] \tag{C.51}$$

$$\xi \varrho_t^I x_t^I \frac{\theta_t}{\theta_t R_t^L + \eta} + \xi \varrho_t^E x_t^E \frac{\theta_t}{\theta_t R_t^L + \eta} = \mathbb{E}_t q_{t,t+1} p_{k,t} L_{k,t}^I + \mathbb{E}_t q_{t,t+1} p_{k,t} L_{k,t}^E \tag{C.52}$$

where I have imposed $L_t^I = l_t^I$ and $L_t^E = l_t^E$ in a symmetric equilibrium and that the collateral constraint is always binding (holds with equality at all times). Substituting $\theta a^I = l^I R^L$ and $\theta a^E = l^E R^L$ in the first and the second equations respectively, I have

$$\varrho^I = \beta^P\left[pR^L + (1 - p)\frac{\tau\theta a^I}{L^I} - R^D + \gamma^L (1 - \rho_s) \varrho^I\right] \tag{C.53}$$

$$\varrho^E = \beta^P\left[pR^L + (1 - p)\frac{\tau\theta a^E}{L^E} - R^D + \gamma^L (1 - \rho_s) \varrho^E\right] \tag{C.54}$$

Now using $\frac{\theta a^I}{L^I} = R^L$ and $\frac{\theta a^E}{L^E} = R^L$

$$\varrho^I = \beta^P \Big[pR^L + (1-p)\,\tau R^L - R^D + \gamma^L (1-\rho_s)\,\varrho^I \Big] \tag{C.55}$$

$$\varrho^E = \beta^P \Big[pR^L + (1-p)\,\tau R^L - R^D + \gamma^L (1-\rho_s)\,\varrho^E \Big] \tag{C.56}$$

I now have

$$\varrho^I = \beta^P \left[\frac{pR^L + (1-p)\,\tau R^L - R^D}{1-\beta^P\gamma^L (1-\rho_s)} \right] \tag{C.57}$$

and

$$\varrho^E = \beta^P \left[\frac{pR^L + (1-p)\,\tau R^L - R^D}{1-\beta^P\gamma^L (1-\rho_s)} \right] \tag{C.58}$$

from which it's easy to see

$$\varrho^I = \varrho^E \tag{C.59}$$

From bank's third FOC

$$\xi \varrho^I \left(\frac{\frac{\eta}{\theta}}{\theta R^L + \eta} \right) \left(x^I + x^E \right) = -\varpi \beta^P \Big[\left(R^L L^I - \tau\theta a^I \right) + \left(R^L L^E - \tau\theta a^E \right) \Big] \tag{C.60}$$

After subsituting the expressions for x^I and x^E

$$\xi \varrho^I \left(\frac{\frac{\eta}{\theta}}{\theta R^L + \eta} \right) \Big[\left(1-\gamma^L\right) l^I + \left(1-\gamma^L\right) l^E \Big] = -\varpi \beta^P \Big[\left(R^L l^I - \tau R^L l^I \right) + \left(R^L l^E - \tau R^L l^E \right) \Big] \tag{C.61}$$

Simplifying

$$\begin{aligned} \xi \varrho^I \left(\frac{\frac{\eta}{\theta}}{\theta R^L + \eta} \right) \left(1-\gamma^L\right) \left(l^I + l^E \right) &= -\varpi \beta^P \Big[R^L l^I (1-\tau) + R^L l^E (1-\tau) \Big] \\ &= -\varpi \beta^P R^L (1-\tau) \left(l^I + l^E \right) \end{aligned} \tag{C.62}$$

This finally simplifies to

$$\xi \varrho^I \left(\frac{\frac{\eta}{\theta}}{\theta R^L + \eta} \right) \left(1-\gamma^L\right) = -\varpi \beta^P R^L (1-\tau) \tag{C.63}$$

The final FOC of banks optimization problem reads

$$\xi \varrho^I \left(\frac{\theta}{\theta R^L + \eta} \right) \left(x^I + x^E \right) = \beta^P p \left(L^I + L^E \right) \tag{C.64}$$

Rewriting this equation

$$
\begin{aligned}
\xi \varrho^{I}\left(\frac{\theta}{\theta R^{L}+\eta}\right)\left(1-\gamma^{L}\right) &= \beta^{P} p \\
\Rightarrow \xi \mu^{B}\left(1-\gamma^{L}\right) \frac{\theta}{\theta R^{L}+\eta} &= \beta^{P} p \\
\Rightarrow \xi \mu^{B}\left(1-\gamma^{L}\right) \theta &= \beta^{P} p\left(\theta R^{L}+\eta\right) \\
\Rightarrow \theta\left[\xi \mu^{B}\left(1-\gamma^{L}\right)-\beta^{P} p R^{L}\right] &= \beta^{P} p \eta \\
\Rightarrow \theta &= \frac{\beta^{P} p \eta}{\xi \mu^{B}\left(1-\gamma^{L}\right)-\beta^{P} p R^{L}}
\end{aligned} \tag{C.65}
$$

(C.57), (C.63) and (C.65) form a system of 3 equations in 3 unknowns: ϱ^{I}, θ and R^{L}. In order to solve this syetm of equations, I first insert for ϱ^{I} from (C.57) into (C.63) and (C.65). This gives the following system of equation

$$
\begin{aligned}
\xi\left(1-\gamma^{L}\right) \frac{p R^{L}+(1-p) \tau R^{L}-R^{D}}{1-\beta^{P} \gamma^{L}\left(1-\rho_{s}\right)} \frac{\eta}{\theta} &= -\varpi R^{L}(1-\tau)\left(\theta R^{L}+\eta\right) \\
\theta &= \frac{\beta^{P} p \eta}{\xi\left(1-\gamma^{L}\right) \beta^{P} \dfrac{p R^{L}+(1-p) \tau R^{L}-R^{D}}{1-\beta^{P} \gamma^{L}\left(1-\rho_{s}\right)}-\beta^{P} p R^{L}}
\end{aligned}
$$

Plugging the value of θ from the second equation into the first, I obtain the value of R^{L} after which values of ϱ^{I} and θ follow directly. This procedure determines the value of R^{L} exclusively from bank's problem which allows it to be inserted into equations derives from entrepreneur's problems.

Steady state version of aggregate resource constraint (B.37) is

$$
\begin{aligned}
C^{P}+C^{I}+C^{E}+I &= Y \\
\Rightarrow \frac{C^{P}}{Y} &= 1-\frac{C^{I}}{Y}-\frac{C^{E}}{Y}-\delta \frac{K}{Y}
\end{aligned} \tag{C.66}
$$

From steady state value of (B.36)

$$
p=\Xi \tag{C.67}
$$

Combining (C.1), (C.2) and (C.12) gives steady-state equilibrium condition for patient households

$$
\begin{aligned}
\iota &= \lambda^{P} W^{P} \\
\Rightarrow \iota &= \frac{1-\beta^{P} \gamma^{P}}{\left(1-\gamma^{P}\right) C^{P}} \nu(1-\alpha) \frac{Y}{N^{P}} \\
\Rightarrow N^{P} &= \frac{\left(1-\beta^{P} \gamma^{P}\right) \nu(1-\alpha)}{\iota\left(1-\gamma^{P}\right)}\left(\frac{C^{P}}{Y}\right)^{-1}
\end{aligned} \tag{C.68}
$$

Combining (C.5), (C.6) and (C.13) yields steady-state equilibrium condition for impatient households

$$\begin{aligned}
\iota &= \lambda^I W^I \\
\Rightarrow \iota &= \frac{1-\beta^I\gamma^I}{(1-\gamma^I)\,C^I}(1-\nu)(1-\alpha)\frac{Y}{N^I} \\
\Rightarrow N^I &= \frac{\left(1-\beta^I\gamma^I\right)(1-\nu)(1-\alpha)}{\iota\,(1-\gamma^I)}\left(\frac{C^I}{Y}\right)^{-1}
\end{aligned} \tag{C.69}$$

From (B.39), steady state output is

$$\begin{aligned}
Y &= A\Big[\left(N^P\right)^{\nu}\left(N^I\right)^{1-\nu}\Big]^{1-\alpha}\Big[\left(H^E\right)^{\phi}(K)^{1-\phi}\Big]^{\alpha} \\
Y^{1-\alpha} &= A\Big[\left(N^P\right)^{\nu}\left(N^I\right)^{1-\nu}\Big]^{1-\alpha}\left[\left(\frac{H^E}{Y}\right)^{\phi}\left(\frac{K}{Y}\right)^{1-\phi}\right]^{\alpha} \\
Y^{1-\alpha} &= A\Big[\left(N^P\right)^{\nu}\left(N^I\right)^{1-\nu}\Big]^{1-\alpha}\left[\left(\frac{H^E}{Y}\right)^{\phi}\left(\frac{\alpha\,(1-\phi)\,R^L\beta^E}{R^L\left(1-(1-\delta)\,\beta^E\right)-\theta\left(1-\beta^E R^L\right)}\right)^{1-\phi}\right]^{\alpha}
\end{aligned} \tag{C.70}$$

$$Q^H = \frac{\varsigma}{H^P\lambda^P\left(1-\beta^P\right)} \tag{C.71}$$

Dividing impatient household's housing with patient household's housing results in

$$\frac{H^I}{H-H^I-H^E}\frac{C^P}{C^I} = \frac{\left(1-\beta^P\gamma^P\right)\left(1-\beta^P\right)\left(1-\gamma^I\right)R^L}{\left(1-\beta^I\gamma^I\right)\Big[\left(1-\beta^I\right)R^L+\left(1-\beta^I R^L\right)\theta\Big]\left(1-\gamma^P\right)} \tag{C.72}$$

$$Q^H H^I = \frac{\varsigma}{\lambda^I\left(1-\beta^I\right)+\frac{\lambda^I\left(1-\beta^I R^L\right)}{R^L}\theta} \tag{C.73}$$

$$\begin{aligned}
Q^H &= \frac{\varsigma}{H^I\Big[\lambda^I\left(1-\beta^I\right)+\frac{\lambda^I\left(1-\beta^I R^L\right)}{R^L}\theta\Big]} \\
&= \frac{\varsigma R^L\left(1-\gamma^I\right)C^I}{H^I\left(1-\beta^I\gamma^I\right)\Big[R^L\left(1-\beta^I\right)+\left(1-\beta^I R^L\right)\theta\Big]}
\end{aligned} \tag{C.74}$$

$$\frac{C^I}{Y} = (1-\nu)(1-\alpha)+\Big[1-pR^L-(1-p)\,\tau R^L\Big]\frac{\theta}{R^L}\frac{\varsigma R^L\left(1-\gamma^I\right)C^I}{\left(1-\beta^I\gamma^I\right)\Big[R^L\left(1-\beta^I\right)+\left(1-\beta^I R^L\right)\theta\Big]} \tag{C.75}$$

$$\frac{H^E}{Y} = \frac{\left(1-\beta^I\gamma^I\right)\Big[R^L\left(1-\beta^I\right)+\left(1-\beta^I R^L\right)\theta\Big]}{\varsigma\left(1-\gamma^I\right)}\left[\frac{\beta\alpha\phi}{\left(1-\beta^I\right)R^L-\theta\left(1-\beta^E R^L\right)}\right]\frac{H^I}{C^I} \tag{C.76}$$

D System of Loglinear Equations

The system of equations log-linearized around their steady state is as below:

D.1 Optimality Conditions of Patient Households

Equations (B.1), (B.2) and (B.4) become

$$\beta^P \gamma^P \mathbb{E}_t \widehat{C}^P_{t+1} - \left(1 + \left(\gamma^P\right)^2 \beta^P\right) \widehat{C}^P_t + \gamma^P \widehat{C}^P_{t-1} = \left(1 - \beta^P \gamma^P\right) \left(1 - \gamma^P\right) \widehat{\lambda}^P \tag{D.1}$$

$$\mathbb{E}_t \widehat{\lambda}^P_{t+1} = \widehat{\lambda}^P_t - \widehat{R}^D_t \tag{D.2}$$

$$\widehat{\iota}_t = \widehat{\lambda}^P_t + \widehat{W}_t \tag{D.3}$$

Log-linearization of (B.3) yields

$$\left(1 - \beta^P\right) \left(\widehat{\varsigma}_t - \widehat{H}^P_t\right) + \beta^P \mathbb{E}_t \left[\widehat{\lambda}^P_{t+1} + \widehat{Q}^H_{t+1}\right] = \widehat{\lambda}^P_t + \widehat{Q}^H_t \tag{D.4}$$

D.2 Optimality Conditions of Impatient Households

From (B.5), (B.6) and (B.7), I obtain

$$\beta^I \gamma^I \mathbb{E}_t \widehat{C}^I_{t+1} - \left(1 + \left(\gamma^I\right)^2 \beta^I\right) \widehat{C}^I_t + \gamma^I \widehat{C}^I_{t-1} = \left(1 - \beta^I \gamma^I\right) \left(1 - \gamma^I\right) \widehat{\lambda}^I \tag{D.5}$$

$$\left(1 - \beta^I\right) \left(\widehat{\varsigma}_t - \widehat{H}^I_t\right) + \beta^I \mathbb{E}_t \left(\widehat{\lambda}^I_{t+1} + \widehat{Q}^H_{t+1}\right) = \widehat{\lambda}^I_t + \widehat{Q}^H_t \tag{D.6}$$

and

$$\widehat{\iota}_t = \widehat{\lambda}^I_t + \widehat{W}^I_t \tag{D.7}$$

respectively. Besides, (B.9) and (B.10) give

$$\widehat{s}^I_t = \rho_s \widehat{s}^I_{t-1} + (1 - \rho_s)\, \widehat{l}^I_t \tag{D.8}$$

and

$$\widehat{x}^I_t = \frac{\widehat{l}^I_t}{1 - \gamma^L} - \frac{\gamma^L \widehat{s}^I_{t-1}}{1 - \gamma^L} \tag{D.9}$$

The budget constraint (B.12) becomes

$$
\begin{aligned}
C^I\widehat{C}_t^I + R^L l^I\left(\widehat{R}_{t-1}^L + \widehat{l}_{t-1}^I\right) = {} & W^I N^I\left(\widehat{W}_t^I + \widehat{N}_t^I\right) - Q^H H^I\left(\widehat{H}_t^I - \widehat{H}_{t-1}^I\right) + x^I\widehat{x}_t^I + \gamma^L s^I\widehat{s}_{t-1}^I \\
& + R^L L^I\left(\widehat{R}_{t-1}^L + \widehat{L}_{t-1}^I\right) - \tau a^I\widehat{a}_{t-1}^I - pR^L L^I\left(\widehat{p}_{t-1} + \widehat{R}_{t-1}^L + \widehat{L}_{t-1}^I\right) \\
& + \tau p a^I\left(\widehat{p}_{t-1} + \widehat{a}_{t-1}^I\right) \qquad \text{(D.10)}
\end{aligned}
$$

From borrowing constraint (B.13) and the definition of total assets (B.14), I get

$$
\widehat{l}_t^I = \widehat{\theta}_t + \widehat{a}_t^I - \widehat{R}_t^L \qquad \text{(D.11)}
$$

and

$$
\widehat{a}_t^I = \widehat{Q}_{t+1}^H + \widehat{H}_t^I \qquad \text{(D.12)}
$$

From (B.8), I have

$$
\widehat{\lambda}_t^I = \widehat{R}_t^L + \beta^I R^L \mathbb{E}_t\widehat{\lambda}_{t+1}^I + \left(1 - \beta^I R^L\right)\widehat{\mu}_t^I \qquad \text{(D.13)}
$$

(B.6) yields

$$
\begin{aligned}
\left(\widehat{\lambda}_t^I + \widehat{Q}_t^H\right) = {} & \beta^I\mathbb{E}_t\left(\widehat{\lambda}_{t+1}^I + \widehat{Q}_{t+1}^H\right) + \left(\frac{1}{R^L} - \beta^I\right)\theta\left[\widehat{\mu}_t^I + \widehat{\theta}_t + \mathbb{E}_t\widehat{Q}_{t+1}^H\right] \\
& + \left[\left(1 - \beta^I\right) + \theta\left(\frac{1}{R^L} - \beta^I\right)\right]\left(\widehat{\varsigma}_t - \widehat{H}_t\right) \qquad \text{(D.14)}
\end{aligned}
$$

D.3 Optimality Conditions of Entrepreneurs

From (B.15) and (B.16), I have

$$
\beta^E\gamma^E\mathbb{E}_t\widehat{C}_{t+1}^E - \left(1 + \left(\gamma^E\right)^2\beta^E\right)\widehat{C}_t^E + \gamma^E\widehat{C}_{t-1}^E = \left(1 - \beta^E\gamma^E\right)\left(1 - \gamma^E\right)\widehat{\lambda}_t^E \qquad \text{(D.15)}
$$

and

$$
\widehat{\lambda}_t^E = \widehat{R}_t^L + \beta^E R^L \mathbb{E}_t\widehat{\lambda}_{t+1}^E + \left(1 - \beta^E R^L\right)\widehat{\mu}_t^E \qquad \text{(D.16)}
$$

Equations (B.17) and (B.18) yield

$$
\widehat{W}_t^I = \widehat{Y}_t - \widehat{N}_t^I \qquad \text{(D.17)}
$$

and

$$
\widehat{W}_t^P = \widehat{Y}_t - \widehat{N}_t^P \qquad \text{(D.18)}
$$

From (B.19), I derive

$$\left(\widehat{\lambda}_t^E + \widehat{Q}_t^H\right) = \beta^E \mathbb{E}_t \left(\widehat{\lambda}_{t+1}^E + \widehat{Q}_{t+1}^H\right) + \left(\frac{1}{R^L} - \beta^E\right) \theta \mathbb{E}_t \left(\widehat{\mu}_t^E + \widehat{\theta}_t + \widehat{Q}_{t+1}^H\right)$$
$$+ \left[\left(1-\beta^E\right) - \theta\left(\frac{1}{R^L} - \beta^E\right)\right] \mathbb{E}_t \left[\widehat{\lambda}_{t+1}^E + \widehat{Y}_{t+1} - \widehat{H}_t^E\right] \tag{D.19}$$

Equation (B.20) becomes

$$\widehat{Q}_t^K = \left[1 - \beta^E\left(1-\delta\right) - \theta\left(\frac{1}{R^L} - \beta^E\right)\right] \mathbb{E}_t \left[\widehat{\lambda}_{t+1}^E - \lambda_t^E + \widehat{Y}_{t+1} - K_t\right]$$
$$+ \beta^E\left(1-\delta\right)\mathbb{E}_t\left(\widehat{Q}_{t+1}^K + \widehat{\lambda}_{t+1}^E - \widehat{\lambda}_t^E\right) + \left(1 - \beta^E R^L\right)\frac{1}{R^L}\theta\mathbb{E}_t\left[\widehat{\mu}_t^E - \widehat{\lambda}_t^E + \widehat{\theta}_t + \widehat{Q}_{t+1}^K\right] \tag{D.20}$$

Equation (B.21) is approximated as

$$\widehat{Q}_t^K = \left(1+\beta^E\right)\Omega\widehat{I}_t - \beta^E\Omega\mathbb{E}_t\widehat{I}_{t+1} - \Omega\widehat{I}_{t-1} \tag{D.21}$$

From (B.22) and (B.23), I get

$$\widehat{s}_t^E = \rho_s \widehat{s}_{t-1}^E + \left(1-\rho_s\right)\widehat{l}_t^E \tag{D.22}$$

$$\widehat{x}_t^E = \frac{\widehat{l}_t^E}{1-\gamma^L} - \frac{\gamma^L \widehat{s}_{t-1}^E}{1-\gamma^L} \tag{D.23}$$

Entrepreneurs' budget contraint (B.25) becomes

$$C^E\widehat{C}_t^E + R^L l^E\left(\widehat{R}_{t-1}^L + \widehat{l}_{t-1}^E\right) = Y\widehat{Y}_t - W^P N^P\left(\widehat{W}_t^P + \widehat{N}_t^P\right) - W^I N^I\left(\widehat{W}_t^I + \widehat{N}_t^I\right) - I\widehat{I}_t$$
$$- Q^H H^E\left(\widehat{H}_t^E - \widehat{H}_{t-1}^E\right) + x^E\widehat{x}_t^E + \gamma^L s^E\widehat{s}_{t-1}^E + R^L L^E\left(\widehat{R}_{t-1}^L + \widehat{L}_{t-1}^E\right)$$
$$- \tau a^E\widehat{a}_{t-1}^E - pR^L L^E\left(\widehat{p}_{t-1} + \widehat{R}_{t-1}^L + \widehat{L}_{t-1}^E\right) + \tau p a^E\left(\widehat{p}_{t-1} + \widehat{a}_{t-1}^E\right) \tag{D.24}$$

The borrowing constraint (B.26) yields

$$\widehat{l}_t^E = \widehat{\theta}_t + \widehat{a}_t^E - \widehat{R}_t^L \tag{D.25}$$

The definition of entrepreneurs' total assets (B.27) gives

$$\widehat{a}_t^E = \frac{Q^H H^E}{Q^H H^E + Q^K K}\mathbb{E}_t\left(\widehat{Q}_{t+1}^H + \widehat{H}_t^E\right) + \frac{Q^K K}{Q^H H^E + Q^K K}\mathbb{E}_t\left(\widehat{Q}_{t+1}^K + \widehat{K}_t\right) \tag{D.26}$$

Linearized version of (B.28) is

$$\widehat{\kappa}_t^E = \widehat{\lambda}_t^E + \widehat{Q}_t^K \tag{D.27}$$

D.4 Optimality Conditions of Banks

From (B.29) and (B.30), I obtain

$$\frac{\varrho^I}{\beta^P}\widehat{\varrho}_t^I - \varrho^I\gamma^L(1-\rho_s)\mathbb{E}_t\varrho_{t+1}^I = \left[pR^L + (1-p)\tau R^L - R^D + \varrho^I\gamma^L(1-\rho_s)\right]\mathbb{E}_t\widehat{q}_{t,t+1} + pR^L\left(\widehat{p}_t + \widehat{R}_t^L\right) - R^D\widehat{R}_t^D + (1-p)\tau R^L\widehat{R}_t^L - p\tau R^L\widehat{p}_t \tag{D.28}$$

and

$$\frac{\varrho^E}{\beta^P}\widehat{\varrho}_t^E - \varrho^E\gamma^L(1-\rho_s)\mathbb{E}_t\varrho_{t+1}^E = \left[pR^L + (1-p)\tau R^L - R^D + \varrho^E\gamma^L(1-\rho_s)\right]\mathbb{E}_t\widehat{q}_{t,t+1} + pR^L\left(\widehat{p}_t + \widehat{R}_t^L\right) - R^D\widehat{R}_t^D + (1-p)\tau R^L\widehat{R}_t^L - p\tau R^L\widehat{p}_t \tag{D.29}$$

Equation (B.31) becomes

$$\begin{aligned}\frac{\eta\xi\varrho^I x^I}{\theta}\left(\widehat{\varrho}_t^I + \widehat{x}_t^I - \widehat{\theta}_t\right) + \frac{\eta\xi\varrho^E x^E}{\theta}\left(\widehat{\varrho}_t^E + \widehat{x}_t^E - \widehat{\theta}_t\right) &= -\varpi\beta^P\left(R^L\right)^2 L^I\theta\left(2\widehat{R}_t^L + \widehat{L}_t^I + \widehat{\theta}_t + \mathbb{E}_t\widehat{q}_{t,t+1}\right)\\ &- \eta\varpi\beta^P R^L L^I\left(\widehat{R}_t^L + \widehat{L}_t^I + \mathbb{E}_t\widehat{q}_{t,t+1}\right)\\ &+ \varpi\tau\beta^P\alpha\theta^2 R^L\left(\widehat{a}_t^I + 2\widehat{\theta}_t + \widehat{R}_t^L + \mathbb{E}_t\widehat{q}_{t,t+1}\right)\\ &+ \eta\varpi\tau\beta^P\theta a^I\left(\widehat{a}_t^I + \widehat{\theta}_t + \mathbb{E}_t\widehat{q}_{t,t+1}\right)\\ &- \varpi\beta^P\left(R^L\right)^2 L^E\theta\left(2\widehat{R}_t^L + \widehat{L}_t^E + \widehat{\theta}_t + \mathbb{E}_t\widehat{q}_{t,t+1}\right)\\ &- \eta\varpi\beta^P R^L L^E\left(\widehat{R}_t^L + \widehat{L}_t^E + \mathbb{E}_t\widehat{q}_{t,t+1}\right)\\ &+ \varpi\tau\beta^P\alpha\theta^2 R^L\left(\widehat{a}_t^E + 2\widehat{\theta}_t + \widehat{R}_t^L + \mathbb{E}_t\widehat{q}_{t,t+1}\right)\\ &+ \eta\varpi\tau\beta^P\theta a^E\left(\widehat{a}_t^E + \widehat{\theta}_t + \mathbb{E}_t\widehat{q}_{t,t+1}\right)\end{aligned} \tag{D.30}$$

From (B.32), I get

$$\begin{aligned}\xi\varrho^I x^I\theta\left(\widehat{\varrho}_t^I + \widehat{x}_t^I + \widehat{\theta}_t\right) + \xi\varrho^E x^E\theta\left(\widehat{\varrho}_t^E + \widehat{x}_t^E + \widehat{\theta}_t\right) &= \theta\beta^P R^L pL^I\left(\widehat{\theta}_t + \widehat{R}_t^L + \widehat{p}_t + \widehat{L}_t^I + \mathbb{E}_t\widehat{q}_{t,t+1}\right)\\ &+ \eta\beta^P pL^I\left(\widehat{p}_t + \widehat{L}_t^I + \mathbb{E}_t\widehat{q}_{t,t+1}\right)\\ &+ \theta\beta^P R^L pL^E\left(\widehat{\theta}_t + \widehat{R}_t^L + \widehat{p}_t + \widehat{L}_t^E + \mathbb{E}_t\widehat{q}_{t,t+1}\right)\\ &+ \eta\beta^P pL^E\left(\widehat{p}_t + \widehat{L}_t^E + \mathbb{E}_t\widehat{q}_{t,t+1}\right)\end{aligned} \tag{D.31}$$

Linearized versions of (B.35) and (B.36) are

$$\widehat{q}_{t,t+1} = \widehat{\lambda}_{t+1}^P - \widehat{\lambda}_t^P \tag{D.32}$$

and

$$p\widehat{p}_t = \varpi\theta\widehat{\theta}_t \tag{D.33}$$

Equation (B.34) gives

$$\widehat{D}_t = \frac{L^I}{L^I + L^E}\widehat{L}_t^I + \frac{L^E}{L^I + L^E}\widehat{L}_t^E \tag{D.34}$$

D.5 Market Clearing and Resource Constraints

Equations (B.37) and (B.38) yield

$$\widehat{Y}_t = \frac{C^P}{C}\widehat{C}_t^P + \frac{C^I}{C}\widehat{C}_t^I + \frac{C^E}{Y}\widehat{C}_t^E + \frac{I}{Y}\widehat{I}_t \tag{D.35}$$

and

$$H^P\widehat{H}_t^P + H^I\widehat{H}_t^I + H^E\widehat{H}_t^E = 0 \tag{D.36}$$

From (B.39) we have

$$\widehat{Y}_t = \widehat{A}_t + \nu\left(1-\alpha\right)\widehat{N}_t^P + \left(1-\nu\right)\left(1-\alpha\right)\widehat{N}_t^I + \alpha\phi\widehat{H}_{t-1}^E + \alpha\left(1-\phi\right)\widehat{K}_{t-1} \tag{D.37}$$

Equation (B.40) gives

$$\widehat{K}_t = \left(1-\delta\right)\widehat{K}_{t-1} + \delta\widehat{I}_t \tag{D.38}$$

E Market Clearing

As mentioned in the main text, two types of transfers Ψ_t^I and Φ_t^I, and Ψ_t^E and Φ_t^E to both impatient households and entrepreneurs are needed to ensure all markets clear. This section demonstrates this and shows the derivation of the expression for Ψ_t^I and Ψ_t^E. The steps in this section closely follow Ravn (2016). Let's start by adding together the budget constraints of patient households, impatient households and entrepreneurs. We sum over both types of households and entrepreneurs, respectively:

$$\begin{aligned}
&\int_0^1 \left(C_{i,t}^P + Q_t^H\left(H_{i,t}^P - H_{i,t-1}^P\right) + \int_0^1 D_{ik,t}\mathrm{d}k\right)\mathrm{d}i + \int_0^1\left(C_{m,t}^I + R_{t-1}^L\int_0^1 l_{mk,t-1}^I\mathrm{d}k\right)\mathrm{d}m \\
&\qquad + \int_0^1\left(C_{j,t}^E + R_{t-1}^L\int_0^1 l_{jk,t-1}^E\mathrm{d}k\right)\mathrm{d}j = \int_0^1\left(W_t^P N_{i,t}^P + \int_0^1 \Pi_{ik,t}\mathrm{d}k + R_{t-1}^D\int_0^1 D_{ik,t-1}\mathrm{d}k\right)\mathrm{d}i \\
&\qquad + \int_0^1\left(W_t^I N_{m,t}^I - Q_t^H\left(H_{m,t}^I - H_{m,t-1}^I\right) + x_{m,t}^I + \Phi_t^I + \Psi_t^I\right)\mathrm{d}m \\
&\qquad + \int_0^1\left(Y_{j,t} - W_t^P N_{j,t}^P - W_t^I N_{j,t}^I - I_{j,t} - Q_t^H\left(H_{j,t}^E - H_{j,t-1}^E\right) + x_{j,t}^E + \Phi_t^E + \Psi_t^E\right)\mathrm{d}j
\end{aligned}$$

After doing the outer integral, I obtain:

$$
\begin{aligned}
&C_t^P + Q_t^H\Big(H_t^P - H_{t-1}^P\Big) + \int_0^1\int_0^1 D_{ik,t}\mathrm{d}i\mathrm{d}k + C_t^I + R_{t-1}^L\int_0^1\int_0^1 l_{mk,t-1}^I\mathrm{d}k\mathrm{d}m \\
&\quad + C_t^E + R_{t-1}^L\int_0^1\int_0^1 l_{jk,t-1}^E\mathrm{d}k\mathrm{d}j = W_t^P N_t^P + \int_0^1\int_0^1 \Pi_{ik,t}\mathrm{d}k\mathrm{d}i + R_{t-1}^D\int_0^1\int_0^1 D_{ik,t-1}\mathrm{d}k\mathrm{d}i \\
&\quad + W_t^I N_t^I - Q_t^H\Big(H_t^I - H_{t-1}^I\Big) + \int_0^1 x_{m,t}^I\mathrm{d}m + \int_0^1 \Phi_t^I\mathrm{d}m + \int_0^1 \Psi_t^I\mathrm{d}m \\
&\quad + Y_t - W_t^P N_t^P - W_t^I N_t^I - I_t - Q_t^H\left(H_t^E - H_{t-1}^E\right) + \int_0^1 x_{j,t}^E\mathrm{d}j + \int_0^1 \Phi_t^E\mathrm{d}j + \int_0^1 \Psi_t^E\mathrm{d}j
\end{aligned}
$$

Using housing market clearing condition, rewrite the above expression:

$$
\begin{aligned}
&C_t^P + C_t^I + C_t^E + I_t - Y_t + Q_t\bigg(\Big(H - H_t^I - H_t^E\Big) - \Big(H - H_{t-1}^I - H_{t-1}^E\Big)\bigg) + Q_t^H\Big(H_t^I - H_{t-1}^I\Big) \\
&\quad + Q_t^H\Big(H_t^E - H_{t-1}^E\Big) + \int_0^1\int_0^1 D_{ik,t}\mathrm{d}i\mathrm{d}k + R_{t-1}^L\int_0^1\int_0^1 l_{mk,t-1}^I\mathrm{d}k\mathrm{d}m + R_{t-1}^L\int_0^1\int_0^1 l_{jk,t-1}^E\mathrm{d}k\mathrm{d}j \\
&\quad = \int_0^1\int_0^1 \Pi_{ik,t}\mathrm{d}k\mathrm{d}i + R_{t-1}^D\int_0^1\int_0^1 D_{ik,t-1}\mathrm{d}k\mathrm{d}i + \int_0^1 x_{m,t}^I\mathrm{d}m + \int_0^1 x_{j,t}^E\mathrm{d}j + \int_0^1 \Phi_t^I\mathrm{d}m + \int_0^1 \Psi_t^I\mathrm{d}m \\
&\quad + \int_0^1 \Phi_t^E\mathrm{d}j + \int_0^1 \Psi_t^E\mathrm{d}j
\end{aligned}
$$

After cancelling terms using the resource constraint, I now plug the expressions for $x_{m,t}^I$, $x_{j,t}^E$, Φ_t^I, Φ_t^E and $\Pi_{k,t}$ from the main text:

$$
\begin{aligned}
\int_0^1\int_0^1 D_{ik,t}\mathrm{d}i\mathrm{d}k &= R_{t-1}^D\int_0^1\int_0^1 D_{ik,t-1}\mathrm{d}k\mathrm{d}i - R_{t-1}^L\int_0^1\int_0^1 l_{mk,t-1}^I\mathrm{d}k\mathrm{d}m - R_{t-1}^L\int_0^1\int_0^1 l_{jk,t-1}^E\mathrm{d}k\mathrm{d}j \\
&+ \int_0^1\left[\int_0^1\Big(l_{mk,t}^I - \gamma^L s_{k,t-1}\Big)^{\frac{\xi-1}{\xi}}\mathrm{d}k\right]^{\frac{\xi}{\xi-1}}\mathrm{d}m + \int_0^1\left[\int_0^1\Big(l_{jk,t}^E - \gamma^L s_{k,t-1}\Big)^{\frac{\xi-1}{\xi}}\mathrm{d}k\right]^{\frac{\xi}{\xi-1}}\mathrm{d}j \\
&+ \gamma^L\int_0^1\int_0^1\frac{\theta_{k,t}}{\theta_t}s_{k,t-1}\mathrm{d}k\mathrm{d}m + \gamma^L\int_0^1\int_0^1\frac{\theta_{k,t}}{\theta_t}s_{k,t-1}\mathrm{d}k\mathrm{d}j + \int_0^1 \Psi_t^I\mathrm{d}m + \Psi_t^E\mathrm{d}j \\
&+ \int_0^1\int_0^1\left(p_{k,t-1}R_{t-1}^L L_{k,t-1}^I + \Big(1 - p_{k,t-1}\Big)\frac{L_{k,t-1}^I}{\int_0^1 L_{k,t-1}^I\mathrm{d}k}\tau\theta_{t-1}a_{t-1}^I - L_{k,t}^I\right)\mathrm{d}k\mathrm{d}i \\
&+ \int_0^1\int_0^1\left(p_{k,t-1}R_{t-1}^L L_{k,t-1}^E + \Big(1 - p_{k,t-1}\Big)\frac{L_{k,t-1}^E}{\int_0^1 L_{k,t-1}^E\mathrm{d}k}\tau\theta_{t-1}a_{t-1}^E - L_{k,t}^E\right)\mathrm{d}k\mathrm{d}i \\
&+ \int_0^1\int_0^1\left(\int_0^1 D_{ik,t}\mathrm{d}i - R_{t-1}^D\int_0^1 D_{ik,t-1}\mathrm{d}i\right)\mathrm{d}k\mathrm{d}i
\end{aligned}
$$

Letting $\xi \to \infty$ and simplifying:

$$\begin{aligned}
\int_0^1 \int_0^1 D_{ik,t} \mathrm{d}i \mathrm{d}k = {} & R^D_{t-1} \int_0^1 \int_0^1 D_{ik,t-1} \mathrm{d}k \mathrm{d}i - R^L_{t-1} \int_0^1 \int_0^1 l^I_{mk,t-1} \mathrm{d}k \mathrm{d}m - R^L_{t-1} \int_0^1 \int_0^1 l^E_{jk,t-1} \mathrm{d}k \mathrm{d}j \\
& + \int_0^1 \int_0^1 \left(l^I_{mk,t} - \gamma^L s_{k,t-1} \right) \mathrm{d}k \mathrm{d}m + \int_0^1 \int_0^1 \left(l^E_{jk,t} - \gamma^L s_{k,t-1} \right) \mathrm{d}k \mathrm{d}j \\
& + \gamma^L \int_0^1 \int_0^1 \frac{\theta_{k,t}}{\theta_t} s_{k,t-1} \mathrm{d}k \mathrm{d}m + \gamma^L \int_0^1 \int_0^1 \frac{\theta_{k,t}}{\theta_t} s_{k,t-1} \mathrm{d}k \mathrm{d}j + \int_0^1 \Psi^I_t \mathrm{d}m + \int_0^1 \Psi^E_t \mathrm{d}j \\
& - R^D_{t-1} \int_0^1 \int_0^1 D_{ik,t-1} \mathrm{d}i \mathrm{d}k \\
& + \int_0^1 \int_0^1 \left(p_{k,t-1} R^L_{t-1} L^I_{k,t-1} + \left(1 - p_{k,t-1} \right) \frac{L^I_{k,t-1}}{\int_0^1 L^I_{k,t-1} \mathrm{d}k} \tau \theta_{t-1} a^I_{t-1} \right) \mathrm{d}k \mathrm{d}i \\
& + \int_0^1 \int_0^1 \left(p_{k,t-1} R^L_{t-1} L^E_{k,t-1} + \left(1 - p_{k,t-1} \right) \frac{L^E_{k,t-1}}{\int_0^1 L^E_{k,t-1} \mathrm{d}k} \tau \theta_{t-1} a^E_{t-1} \right) \mathrm{d}k \mathrm{d}i
\end{aligned}$$

Cancelling terms and further simplifying:

$$\begin{aligned}
\int_0^1 \int_0^1 D_{ik,t} \mathrm{d}i \mathrm{d}k = {} & -R^L_{t-1} \int_0^1 \int_0^1 l^I_{mk,t-1} \mathrm{d}k \mathrm{d}m - R^L_{t-1} \int_0^1 \int_0^1 l^E_{jk,t-1} \mathrm{d}k \mathrm{d}j \\
& + \int_0^1 \int_0^1 \left(l^I_{mk,t} - \gamma^L s_{k,t-1} + \gamma^L \frac{\theta_{k,t}}{\theta_t} s_{k,t-1} \right) \mathrm{d}k \mathrm{d}m \\
& + \int_0^1 \int_0^1 \left(l^E_{jk,t} - \gamma^L s_{k,t-1} + \gamma^L \frac{\theta_{k,t}}{\theta_t} s_{k,t-1} \right) \mathrm{d}k \mathrm{d}j \\
& + \int_0^1 \Psi^I_t \mathrm{d}m + \int_0^1 \Psi^E_t \mathrm{d}j + \int_0^1 \left(1 - p_{k,t-1} \right) \frac{L^I_{k,t-1}}{\int_0^1 L^I_{k,t-1} \mathrm{d}k} \tau \theta_{t-1} a^I_{t-1} \mathrm{d}k \\
& + \int_0^1 \left(1 - p_{k,t-1} \right) \frac{L^E_{k,t-1}}{\int_0^1 L^E_{k,t-1} \mathrm{d}k} \tau \theta_{t-1} a^E_{t-1} \mathrm{d}k + \int_0^1 p_{k,t-1} R^L_{t-1} L^I_{k,t-1} \mathrm{d}k \\
& + \int_0^1 p_{k,t-1} R^L_{t-1} L^E_{k,t-1} \mathrm{d}k
\end{aligned}$$

Cancelling yet more terms and after simplifying more:

$$\begin{aligned}
\int_0^1 \int_0^1 D_{ik,t} \mathrm{d}i \mathrm{d}k = {} & -R^L_{t-1} \int_0^1 L^I_{k,t-1} \mathrm{d}k - R^L_{t-1} \int_0^1 L^E_{k,t-1} \mathrm{d}k + \int_0^1 \int_0^1 l^I_{mk,t} \mathrm{d}k \mathrm{d}m + \int_0^1 \int_0^1 l^I_{jk,t} \mathrm{d}k \mathrm{d}j \\
& + \int_0^1 \Psi^I_t \mathrm{d}m + \int_0^1 \Psi^E_t \mathrm{d}j + \int_0^1 \left(1 - p_{k,t-1} \right) \tau \theta_{t-1} a^I_{t-1} \mathrm{d}k + R^L_{t-1} \int_0^1 p_{k,t-1} L^I_{k,t-1} \mathrm{d}k \\
& + \int_0^1 \left(1 - p_{k,t-1} \right) \tau \theta_{t-1} a^E_{t-1} \mathrm{d}k + R^L_{t-1} \int_0^1 p_{k,t-1} L^E_{k,t-1} \mathrm{d}k
\end{aligned}$$

After moving some terms around:

$$\int_0^1 \left(\int_0^1 D_{ik,t}\mathrm{d}i - L^I_{k,t} - L^E_{k,t}\right)\mathrm{d}k = \int_0^1 \Psi^I_t \mathrm{d}m + \int_0^1 \Psi^E_t \mathrm{d}m + \int_0^1 \left(1 - p_{k,t-1}\right)\tau\theta_{t-1}a^I_{t-1}\mathrm{d}k$$
$$+ \int_0^1 \left(1 - p_{k,t-1}\right)\tau\theta_{t-1}a^E_{t-1}\mathrm{d}k - \int_0^1 \left(1 - p_{k,t-1}\right)R^L_{t-1}L^I_{k,t-1}\mathrm{d}k$$
$$- \int_0^1 \left(1 - p_{k,t-1}\right)R^L_{t-1}L^E_{k,t-1}\mathrm{d}k$$

Due to bank's balance sheet identity, the LHS becomes zero and I now have

$$\int_0^1 \left(1 - p_{k,t-1}\right)R^L_{t-1}L^I_{k,t-1}\mathrm{d}k + \int_0^1 \left(1 - p_{k,t-1}\right)R^L_{t-1}L^E_{k,t-1}\mathrm{d}k - \int_0^1 \left(1 - p_{k,t-1}\right)\tau\theta_{t-1}a^I_{t-1}\mathrm{d}k$$
$$- \int_0^1 \left(1 - p_{k,t-1}\right)\tau\theta_{t-1}a^E_{t-1}\mathrm{d}k = \int_0^1 \Psi^I_t \mathrm{d}m + \int_0^1 \Psi^E_t \mathrm{d}j$$

Finally,

$$\int_0^1 \Psi^I_t \mathrm{d}m = \Psi^I_t = \int_0^1 \left(1 - p_{k,t-1}\right)\left(R^L_{t-1}L^I_{k,t-1} - \tau\theta_{t-1}a^I_{t-1}\right)\mathrm{d}k$$

and

$$\int_0^1 \Psi^E_t \mathrm{d}j = \Psi^E_t = \int_0^1 \left(1 - p_{k,t-1}\right)\left(R^L_{t-1}L^E_{k,t-1} - \tau\theta_{t-1}a^E_{t-1}\right)\mathrm{d}k$$

where Fubini's theorem has been used to switch the order of integrals where necessary.